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PITAGORAS
ACADEMIA

TRIGONOMETRÍA

GRUPO PITÁGORAS

FUNCIONES TRIGONOMÉTRICAS

FUNCIONES TRIGONOMÉTRICAS INVERSAS

FUNCIONES TRIGONOMÉTRICAS INVERSAS

❖ Nociones Previas:

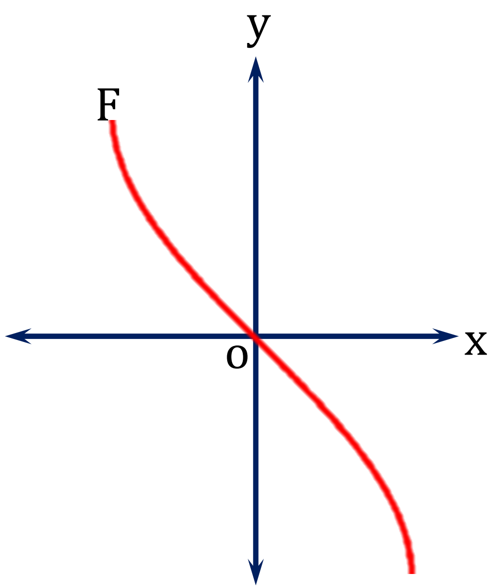
➢ Función Inyectiva:

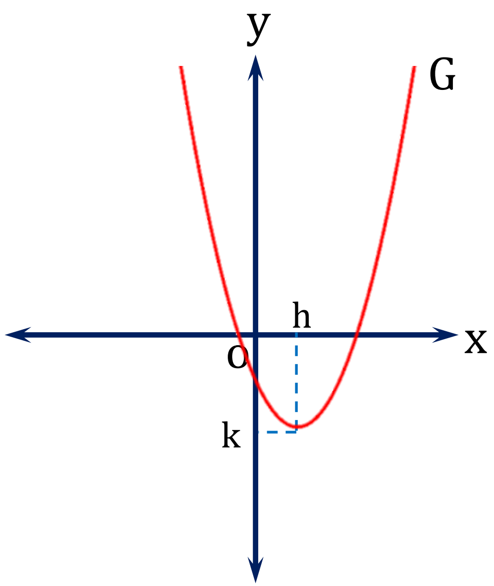
Una función F es inyectiva o univalente si y solo si para todo $x_1, x_2 \in D_f$ se cumple que:

$$F(x_1) = F(x_2) \longrightarrow x_1 = x_2$$

Interpretación Geométrica:

Una función F es inyectiva si cualquier recta horizontal corta a la gráfica de F a lo más en un punto.





De las figuras mostradas se deduce que F es inyectiva, G no es inyectiva en todo su dominio; para que G sea inyectiva se debe redefinir la función, es decir se restringe el dominio, por ejemplo si se escoge el dominio de $G:]-\infty; h]$ entonces G es inyectiva también se puede elegir como dominio $[h; +\infty[$

Nociones Previas:

Función Inversa:

Dada la función $F = \{(x; y)/y = F(x); x \in D_F\}$

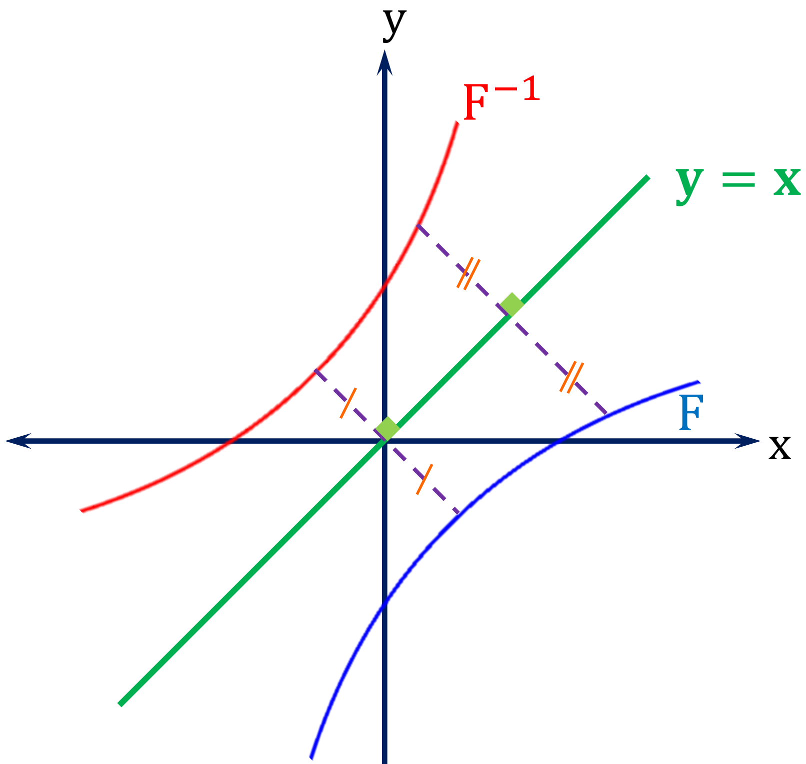
Si F es inyectiva entonces F tiene inversa y se representa por F^*o F^{-1} y se define por: $F^{-1} = \{(y; x)/y = F(x); x \in D_F\}$

También se puede escribir así: $F^{-1} = \{(y; x)/x = F^{-1}(y); y \in R_F\}$

La gráfica de F^{-1} se obtiene reflejando la gráfica de F a través de la recta $y = x$

Se deduce que:

$\boxed{D_{F^{-1}} = R_F}$ $\boxed{R_{F^{-1}} = D_F}$



1.NOTACIÓN:

Sea: $RT(\theta) = N$

Entonces:

$\theta = \text{arcRT}(N)$ ... Notación Francesa

$\theta = RT^{-1}(N)$ ... Notación Inglesa

Aplicación:

- Si: $\text{Sen}\theta = \dfrac{1}{3}$, hallar θ

$$\theta = \text{arcSen}\left(\frac{1}{3}\right)$$

$$\theta = \text{Sen}^{-1}\left(\frac{1}{3}\right)$$

- Si: $\text{Cos}\left(2\theta - \dfrac{2\pi}{3}\right) = \dfrac{2}{5}$, hallar θ

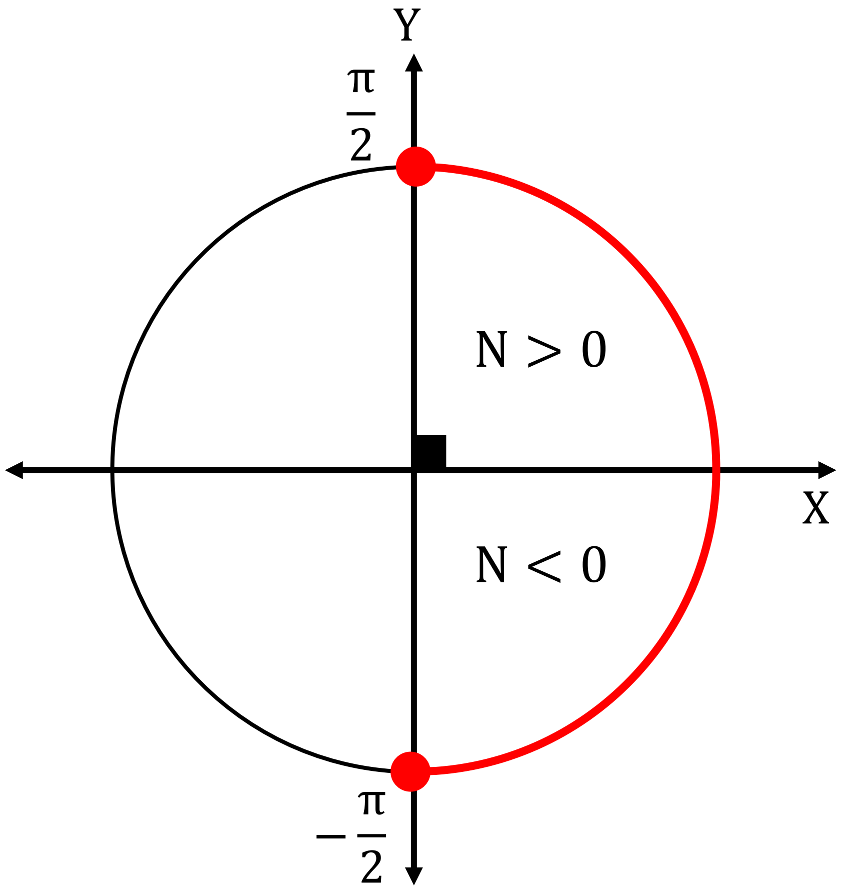
$$2\theta - \frac{2\pi}{3} = \text{arcCos}\left(\frac{2}{5}\right)$$

$$2\theta = \text{arcCos}\left(\frac{2}{5}\right) + \frac{2\pi}{3}$$

$$\therefore \theta = \frac{1}{2}\text{arcCos}\left(\frac{2}{5}\right) + \frac{\pi}{3}$$

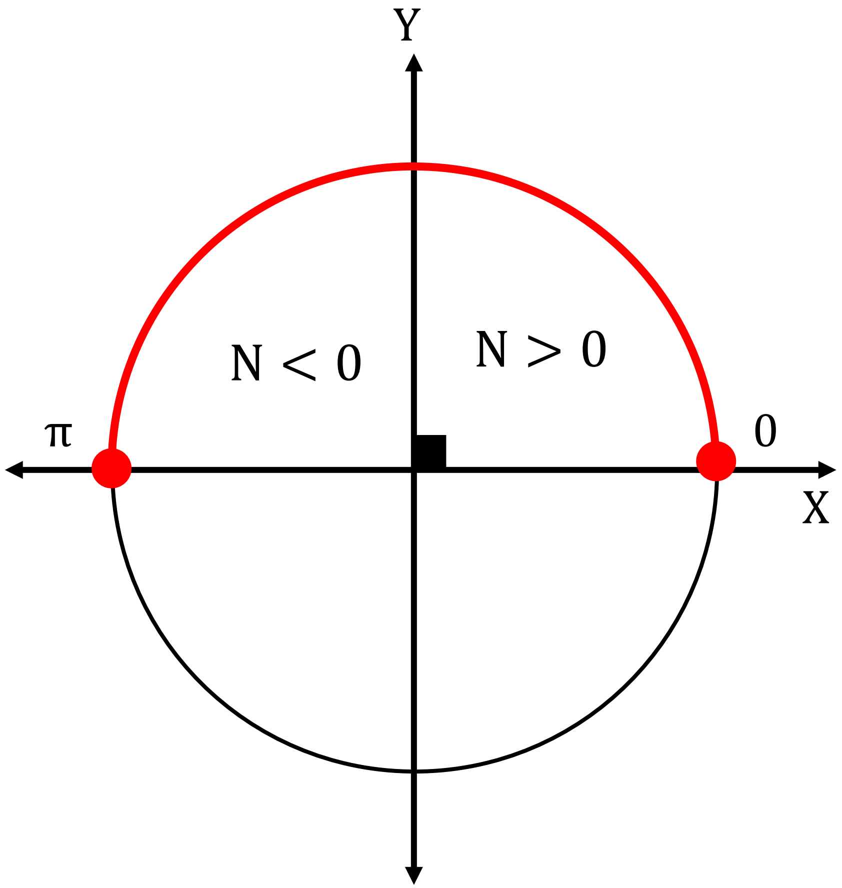
1.1. PROPIEDAD FUNDAMENTAL:

$$\theta = \text{arcSen}N \leftrightarrow \text{Sen}\theta = N \wedge \theta \in \left[-\frac{\pi}{2};\frac{\pi}{2}\right]$$



1.1. PROPIEDAD FUNDAMENTAL:

$$\theta = \text{arcCos}N \leftrightarrow \text{Cos}\theta = N \wedge \theta \in [0; \pi]$$



1.1. PROPIEDAD FUNDAMENTAL:

$$\theta = \operatorname{arcTanN} \leftrightarrow \operatorname{Tan}\theta = N \wedge \theta \in \left]-\frac{\pi}{2}; \frac{\pi}{2}\right[$$

$$\theta = \operatorname{arcCotN} \leftrightarrow \operatorname{Cot}\theta = N \wedge \theta \in \left]0; \pi\right[$$

$$\theta = \operatorname{arcSecN} \leftrightarrow \operatorname{Sec}\theta = N \wedge \theta \in [0; \pi] - \left\{\frac{\pi}{2}\right\}$$

$$\theta = \operatorname{arcCscN} \leftrightarrow \operatorname{Csc}\theta = N \wedge \theta \in \left[-\frac{\pi}{2}; \frac{\pi}{2}\right] - \{0\}$$

FUNCIONES TRIGONOMÉTRICAS INVERSAS

Ejemplo:

Obtenga el valor de las siguientes expresiones:

$$\alpha = \text{arcSen}\frac{1}{2} = \frac{\pi}{6}$$

$$\beta = \text{arcCos}\frac{\sqrt{2}}{2} = \frac{\pi}{4}$$

$$\gamma = \text{arcTan}\left(2 - \sqrt{3}\right) = \frac{\pi}{12}$$

$$\delta = \text{arcCot}(0) = \frac{\pi}{2}$$

$$\varepsilon = \text{arcSec}(2) = \frac{\pi}{3}$$

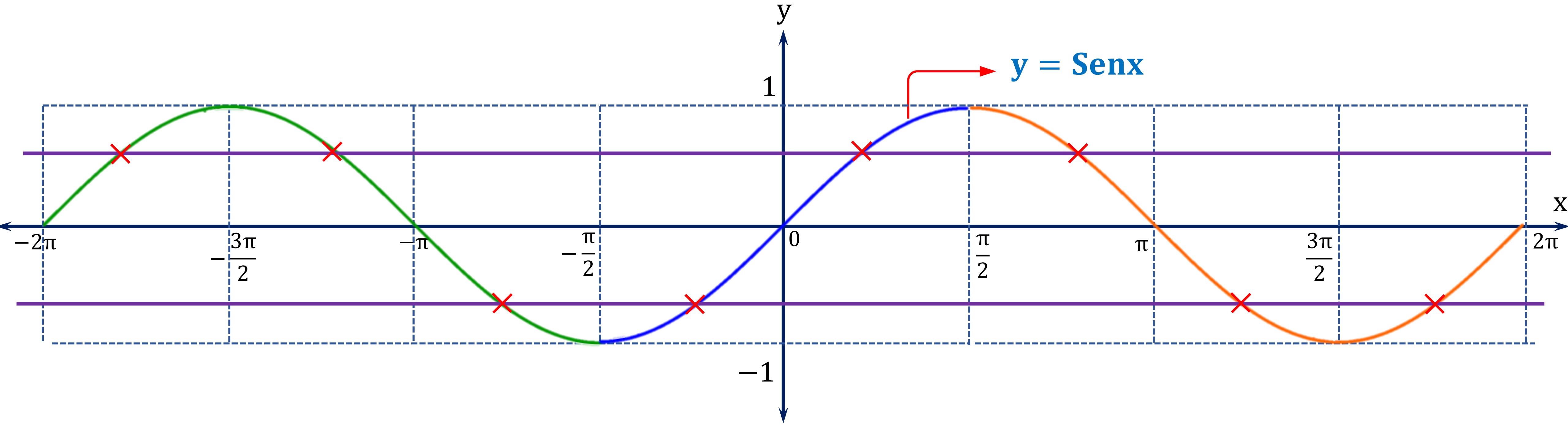
$$\theta = \text{arcCsc}\left(\sqrt{6} - \sqrt{2}\right) = \frac{5\pi}{12}$$

❖ OBSERVACIÓN:

Como las funciones trigonométricas son periódicas, entonces no son inyectivas por lo tanto no tienen inversa en todo su dominio. Para que existan las inversas de dichas funciones, se debe restringir el dominio de modo que sean biyectivas.

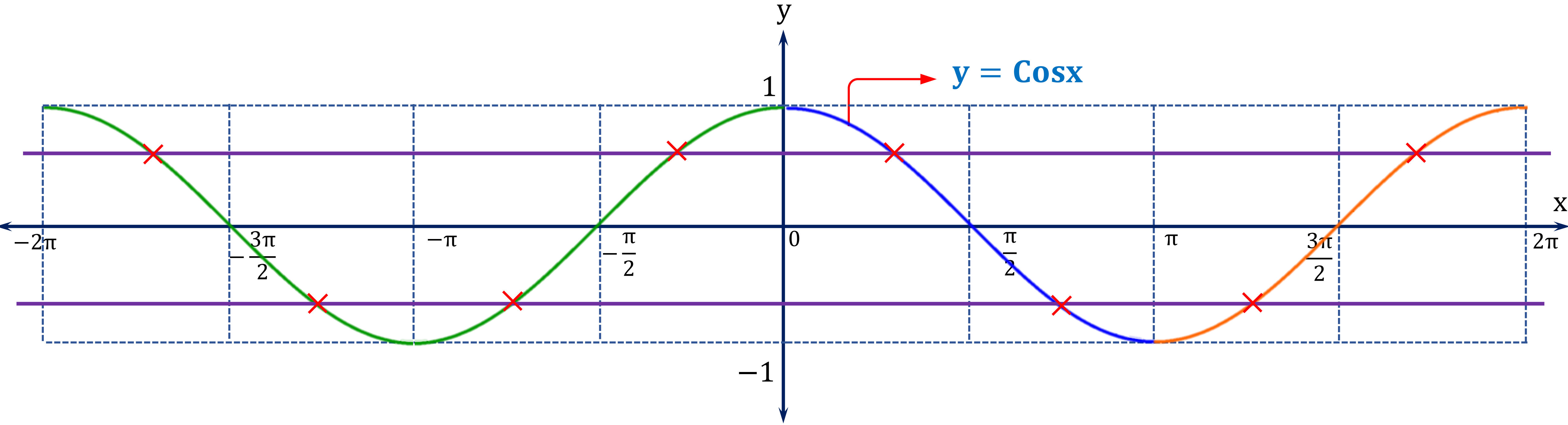
La restricción para las funciones es:

FUNCIÓN SENO:



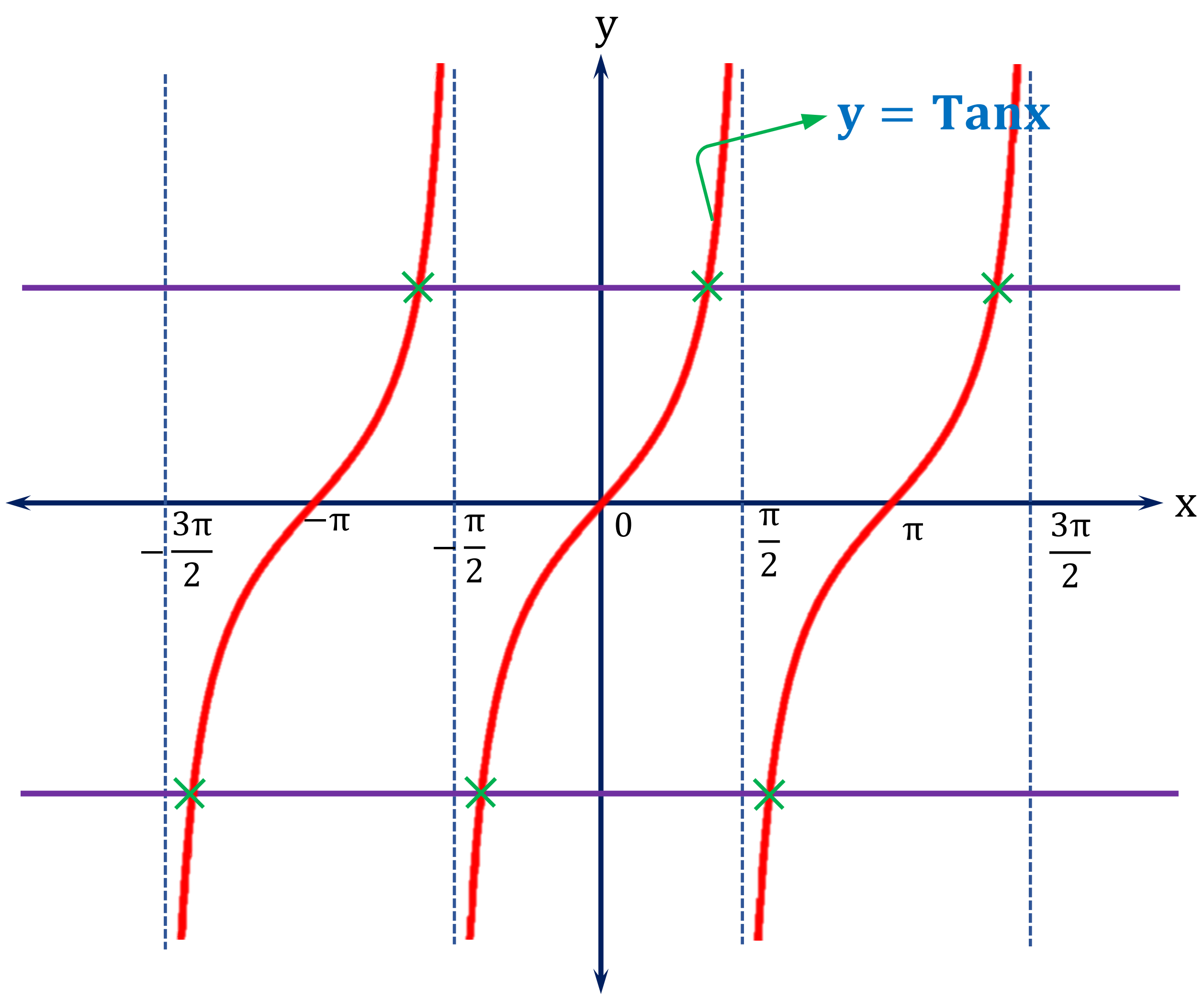
❖ Dominio(restringido): $\left[-\frac{\pi}{2};\frac{\pi}{2}\right]$

FUNCIÓN COSENO:



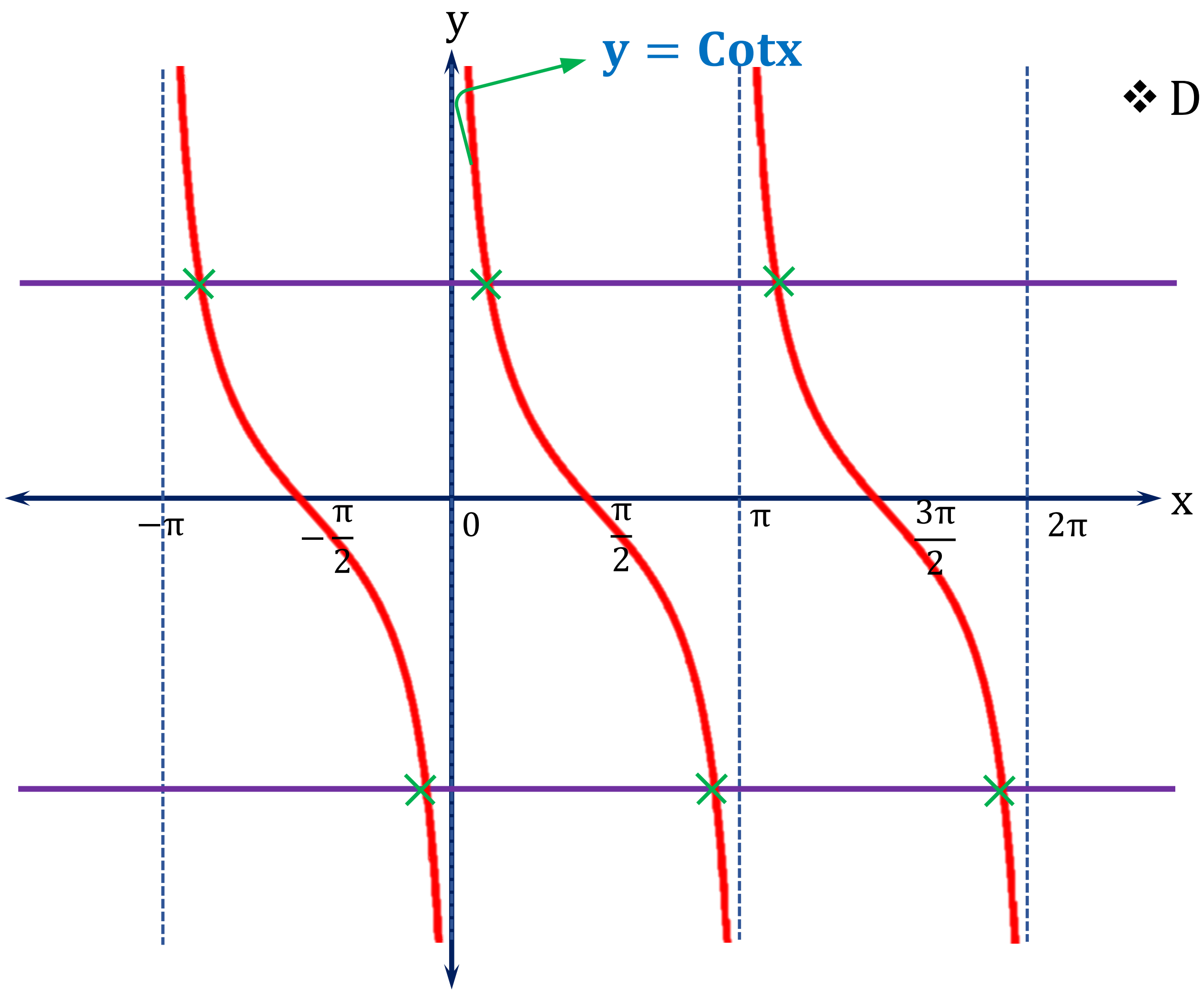
- Dominio(restringido): $[0; \pi]$

FUNCIÓN TANGENTE:



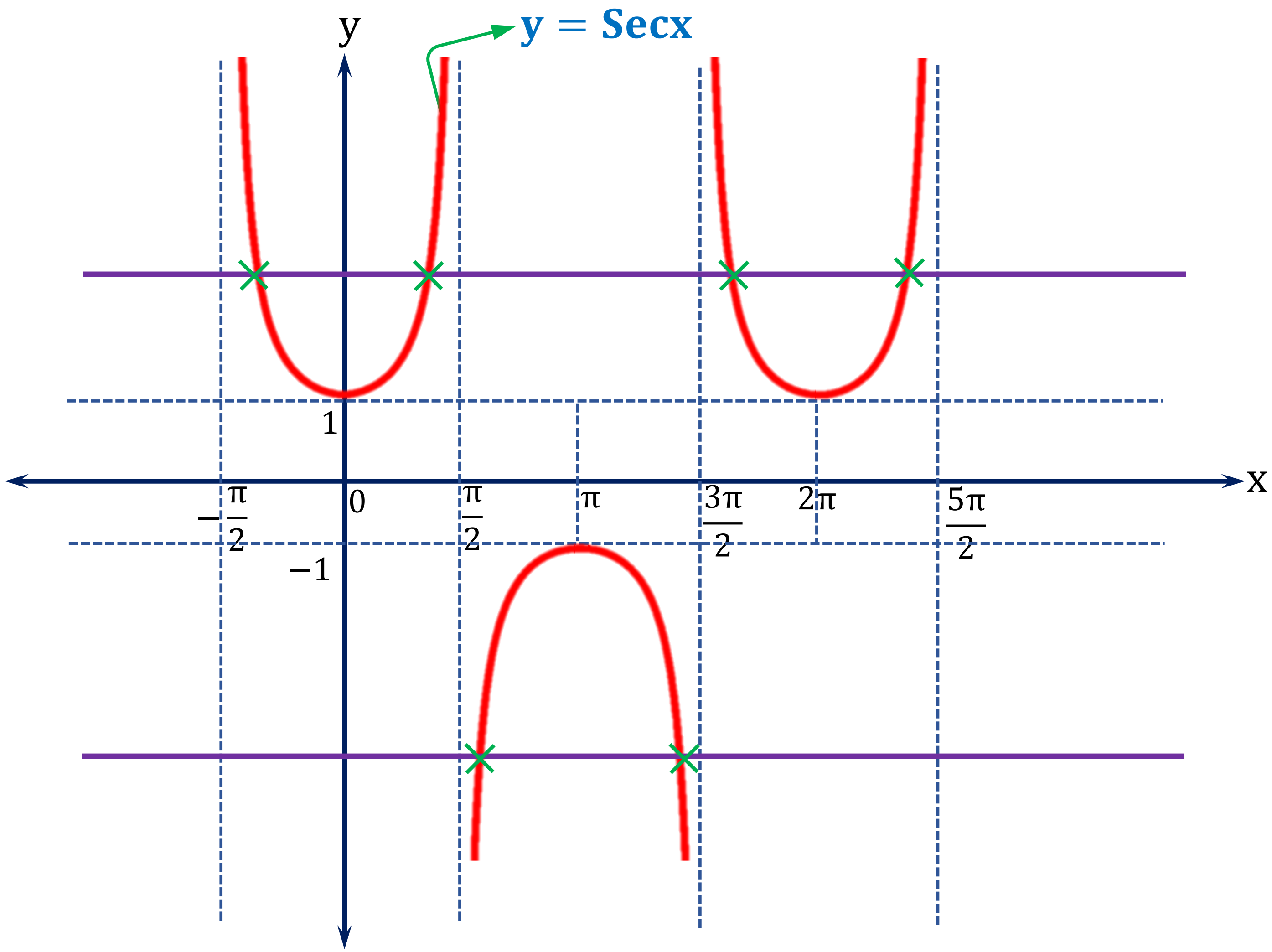
❖ Dominio(restringido): $\left]-\frac{\pi}{2};\frac{\pi}{2}\right[$

FUNCIÓN COTANGENTE:



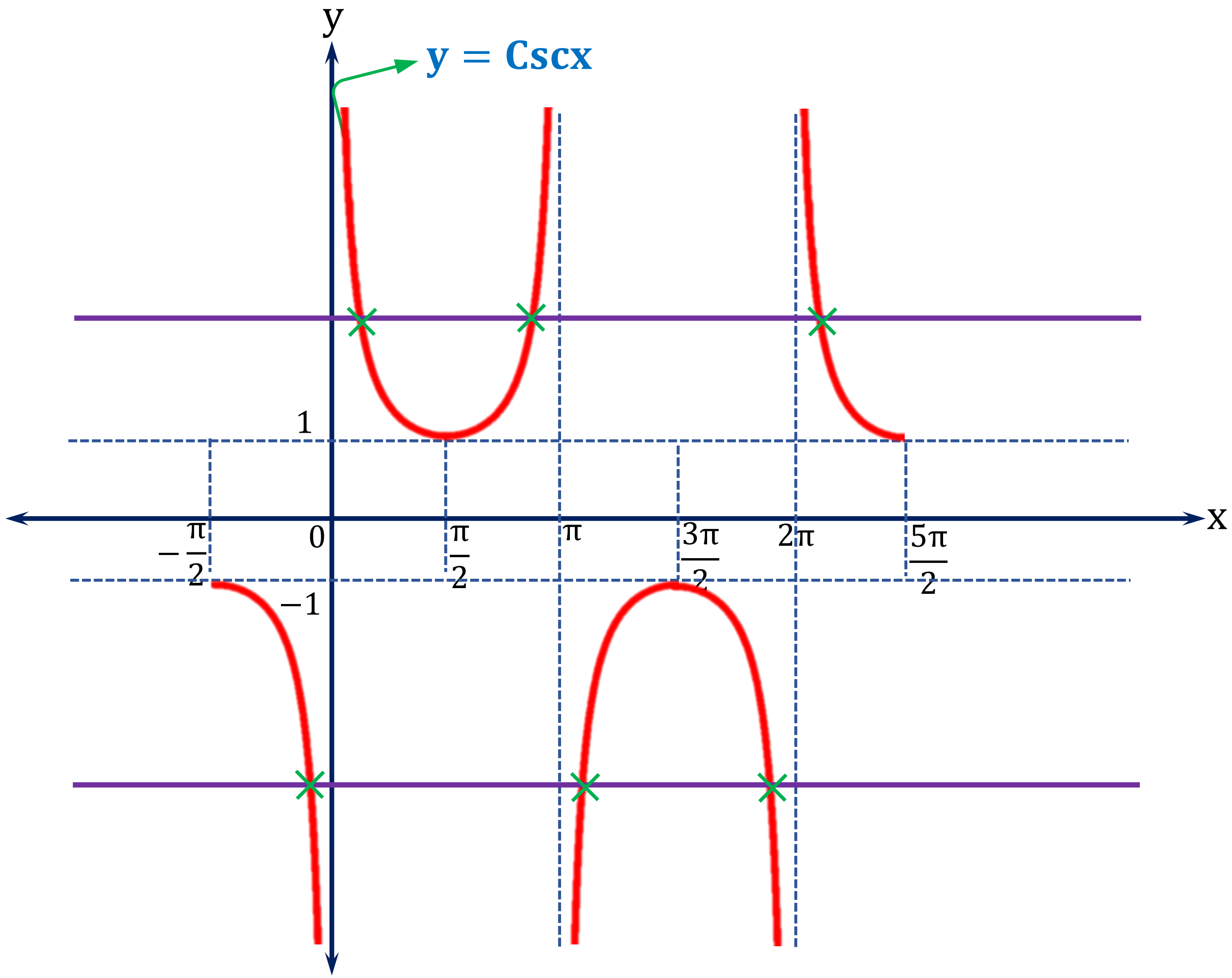
- Dominio(restringido): $]0;\pi[$

FUNCIÓN SECANTE:



❖ Dominio(restringido): $[0;\pi]-\left\{\frac{\pi}{2}\right\}$

FUNCIÓN COSECANTE:



❖ Dominio(restringido): $\left[-\frac{\pi}{2};\frac{\pi}{2}\right]-\{0\}$

❖ OBSERVACIÓN:

Función	Dominio	Rango
y=Senx	$\left[-\frac{\pi}{2};\frac{\pi}{2}\right]$	$[-1;1]$
y=Cosx	$[0;\pi]$	$[-1;1]$
y=Tanx	$\left]-\frac{\pi}{2};\frac{\pi}{2}\right[$	$\mathbb{R}$
y=Cotx	$]0;\pi[$	$\mathbb{R}$
y=Secx	$\left[0;\frac{\pi}{2}\right[\cup\left]\frac{\pi}{2};\pi\right]$	$]-\infty;-1]\cup[1;+\infty[$
y=Cscx	$\left[-\frac{\pi}{2};0\right[\cup\left]0;\frac{\pi}{2}\right]$	$]-\infty;-1]\cup[1;+\infty[$

2.Dominio(D_f) y Rango (R_f):

Función	Dominio	Rango
y=arcSenx	$[-1;1]$	$\left[-\frac{\pi}{2};\frac{\pi}{2}\right]$
y=arcCosx	$[-1;1]$	$[0;\pi]$
y=arcTanx	$\mathbb{R}$	$\left]-\frac{\pi}{2};\frac{\pi}{2}\right[$
y=arcCotx	$\mathbb{R}$	$]0;\pi[$
y=arcSecx	$]-\infty;-1]\cup[1;+\infty[$	$\left[0;\frac{\pi}{2}\right[\cup\left]\frac{\pi}{2};\pi\right]$
y=arcCscx	$]-\infty;-1]\cup[1;+\infty[$	$\left[-\frac{\pi}{2};0\right[\cup\left]0;\frac{\pi}{2}\right]$

Ejemplo:

Determinar el dominio de la función: $F(x) = 2\text{arcSen}\left(\frac{4-5x}{11}\right)$

Resolución:

$$F(x) = 2\text{arcSen}\left(\frac{4-5x}{11}\right)$$

$$-1 \leq \frac{4-5x}{11} \leq 1$$

$$-11 \leq 4-5x \leq 11$$

$$-15 \leq -5x \leq 7$$

$$15 \geq 5x \geq -7$$

$$3 \geq x \geq -\frac{7}{5}$$

$\therefore$ **Dominio:** $\left[-\frac{7}{5}; 3\right]$

Ejemplo:

Determinar el rango: $G(x) = 3\text{arcCsc}(2x) - \dfrac{\pi}{4}$

Resolución:

$$\text{arcCsc}(2x) \rightarrow \text{Rango}: \left[-\frac{\pi}{2}; \frac{\pi}{2}\right] - \{0\}$$

$$3\text{arcCsc}(2x) \rightarrow \text{Rango}: \left[-\frac{3\pi}{2}; \frac{3\pi}{2}\right] - \{0\}$$

$$3\text{arcCsc}(2x) - \frac{\pi}{4} \rightarrow \text{Rango}: \left[-\frac{3\pi}{2} - \frac{\pi}{4}; \frac{3\pi}{2} - \frac{\pi}{4}\right] - \left\{0 - \frac{\pi}{4}\right\}$$

$$\therefore \textbf{Rango}: \left[-\frac{7\pi}{4}; \frac{5\pi}{4}\right] - \left\{-\frac{\pi}{4}\right\}$$

3.PROPIEDADES

❖ $\mathbf{F(F^*(x)) = x \ ; \ x \in Df^*}$

$$\mathrm{Sen(arcSenN) = N \ ; \ -1 \leq N \leq 1}$$

$$\mathrm{Cos(arcCosN) = N \ ; \ -1 \leq N \leq 1}$$

$$\mathrm{Tan(arcTanN) = N \ ; \ -\infty \leq N \leq +\infty}$$

$$\mathrm{Cot(arcCotN) = N \ ; \ -\infty \leq N \leq +\infty}$$

$$\mathrm{Sec(arcSecN) = N \ ; \ N \leq -1 \cup 1 \leq N}$$

$$\mathrm{Csc(arcCscN) = N \ ; \ N \leq -1 \cup 1 \leq N}$$

$$\mathrm{Sen}\left(\mathrm{arcSen}\frac{1}{3}\right) = \frac{1}{3}$$

$$\mathrm{Tan(arcTan4)} = 4$$

$$\mathrm{Sec}\left(\mathrm{arcSec}\frac{1}{2}\right) = \frac{1}{2}$$ ¡Cuidado de hacer esto!

Pues $\left(\frac{1}{2}\right) \notin \mathbb{R} -]-1;1[\longrightarrow \mathrm{Sec}\left(\mathrm{arcSec}\frac{1}{2}\right) : \nexists$

3.PROPIEDADES

❖ $\mathbf{F^{*}(F(x)) = x \ , x \in R^{f}}$

$$\operatorname{arcSen}(\operatorname{Sen}\theta) = \theta \ ; \ -\frac{\pi}{2} \leq \theta \leq \frac{\pi}{2}$$

$$\operatorname{arcCos}(\operatorname{Cos}\theta) = \theta \ ; \ 0 \leq \theta \leq \pi$$

$$\operatorname{arcTan}(\operatorname{Tan}\theta) = \theta \ ; \ -\frac{\pi}{2} < \theta < \frac{\pi}{2}$$

$$\operatorname{arcCot}(\operatorname{Cot}\theta) = \theta \ ; \ 0 < \theta < \pi$$

$$\operatorname{arcSec}(\operatorname{Sec}\theta) = \theta \ ; \ 0 \leq \theta \leq \pi \, , \theta \neq \frac{\pi}{2}$$

$$\operatorname{arcCsc}(\operatorname{Csc}\theta) = \theta \ ; -\frac{\pi}{2} \leq \theta \leq \frac{\pi}{2}, \theta \neq 0$$

$$\operatorname{arcSen}\left(\operatorname{Sen}\frac{\pi}{5}\right) = \frac{\pi}{5}$$

$$\operatorname{arcTan}\left[\operatorname{Tan}\left(-\frac{\pi}{4}\right)\right] = -\frac{\pi}{4}$$

$$\operatorname{arcCsc}\left(\operatorname{Csc}\frac{5\pi}{6}\right) \neq \frac{5\pi}{6}$$

Pues $\left(\frac{5\pi}{6}\right) \notin \left[-\frac{\pi}{2};\frac{\pi}{2}\right] - \{0\}$

3.PROPIEDADES

❖ $\mathbf{arcRT(N) + arcCO - RT(N) = \frac{\pi}{2}}$

$$\mathrm{arcSenN + arcCosN} = \frac{\pi}{2}$$

$$\mathrm{arcTanN + arcCotN} = \frac{\pi}{2}$$

$$\mathrm{arcSecN + arcCscN} = \frac{\pi}{2}$$

$$\mathrm{arcTan}\left(-\sqrt{3}\right) + \mathrm{arcCot}\left(-\sqrt{3}\right) = \frac{\pi}{2}$$

$$\mathrm{arcSen(0) + arcCos}(0) = \frac{\pi}{2}$$

3. PROPIEDADES

- $\mathbf{arcRT(-N); N > 0}$

$$\text{arcSen}(-\text{N}) = -\text{arcSen}(\text{N})$$

$$\text{arcCos}(-\text{N}) = \pi - \text{arcCos}(\text{N})$$

$$\text{arcTan}(-\text{N}) = -\text{arcTan}(\text{N})$$

$$\text{arcCot}(-\text{N}) = \pi - \text{arcCot}(\text{N})$$

$$\text{arcSec}(-\text{N}) = \pi - \text{arcSec}(\text{N})$$

$$\text{arcCsc}(-\text{N}) = -\text{arcCsc}(\text{N})$$

$$\text{arcSen}\left(-\frac{1}{2}\right) = -\text{arcSen}\left(\frac{1}{2}\right)$$

$$\text{arcCos}\left(\frac{-\sqrt{5}-1}{4}\right) = \pi - \text{arcCos}\left(\frac{\sqrt{5}+1}{4}\right)$$

3. PROPIEDADES

- $\mathbf{arcTanA + arcTanB = arcTan\left(\dfrac{A+B}{1-AB}\right) + k\pi}$

Si: $AB > 1 \wedge A > 0 \wedge B > 0 \longrightarrow k = 1$

Si: $AB > 1 \wedge A < 0 \wedge B < 0 \longrightarrow k = -1$

Si: $AB < 1 \longrightarrow k = 0$

- $\mathbf{arcTanA - arcTanB = arcTan\left(\dfrac{A-B}{1+AB}\right)}$

$$\theta = \mathrm{arcTan}\frac{1}{2}$$

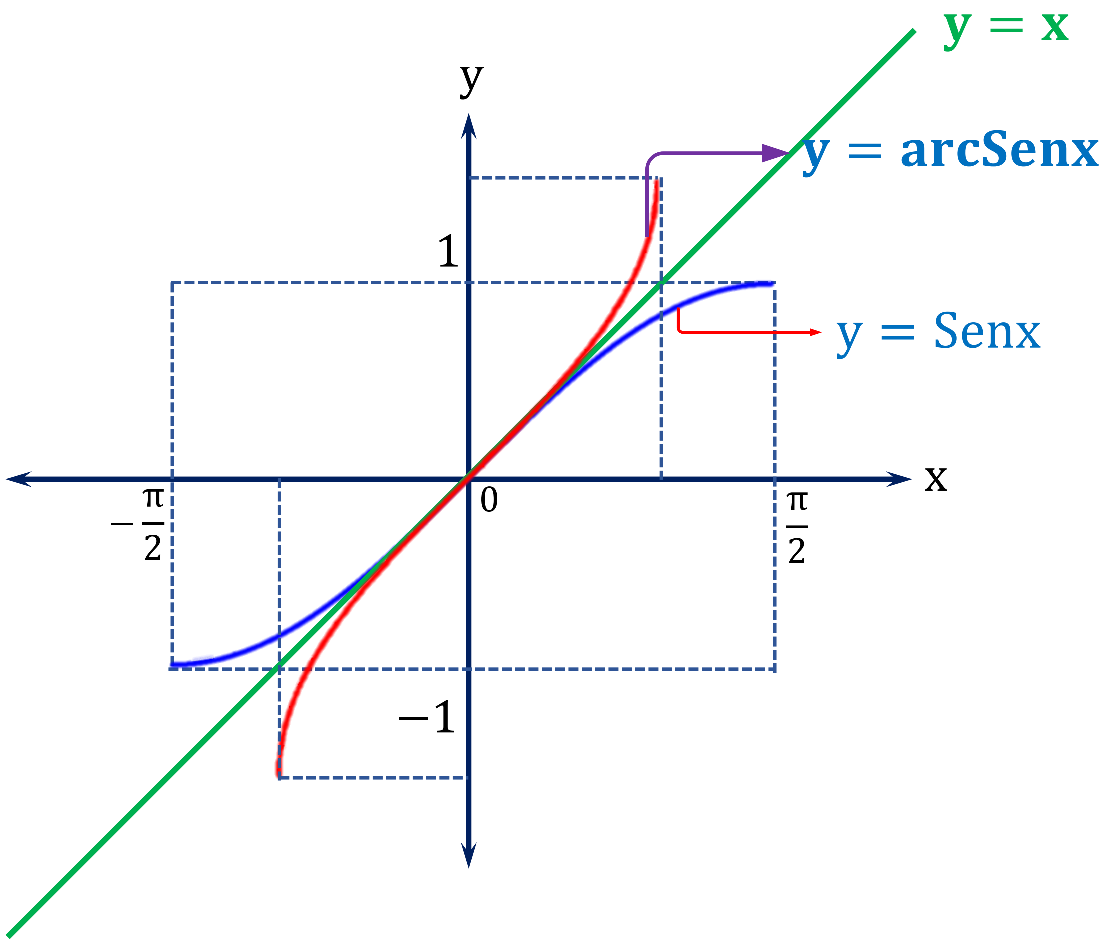
$$\theta = \mathrm{arcTan}\left(\frac{\frac{1}{2}+\frac{1}{3}}{1-\frac{1}{2}\times\frac{1}{3}}\right) + 0\pi$$

$$\theta = \mathrm{arcTan}\left(\frac{\frac{5}{6}}{1-\frac{1}{6}}\right)$$

$$\theta = \mathrm{arcTan}(1)$$

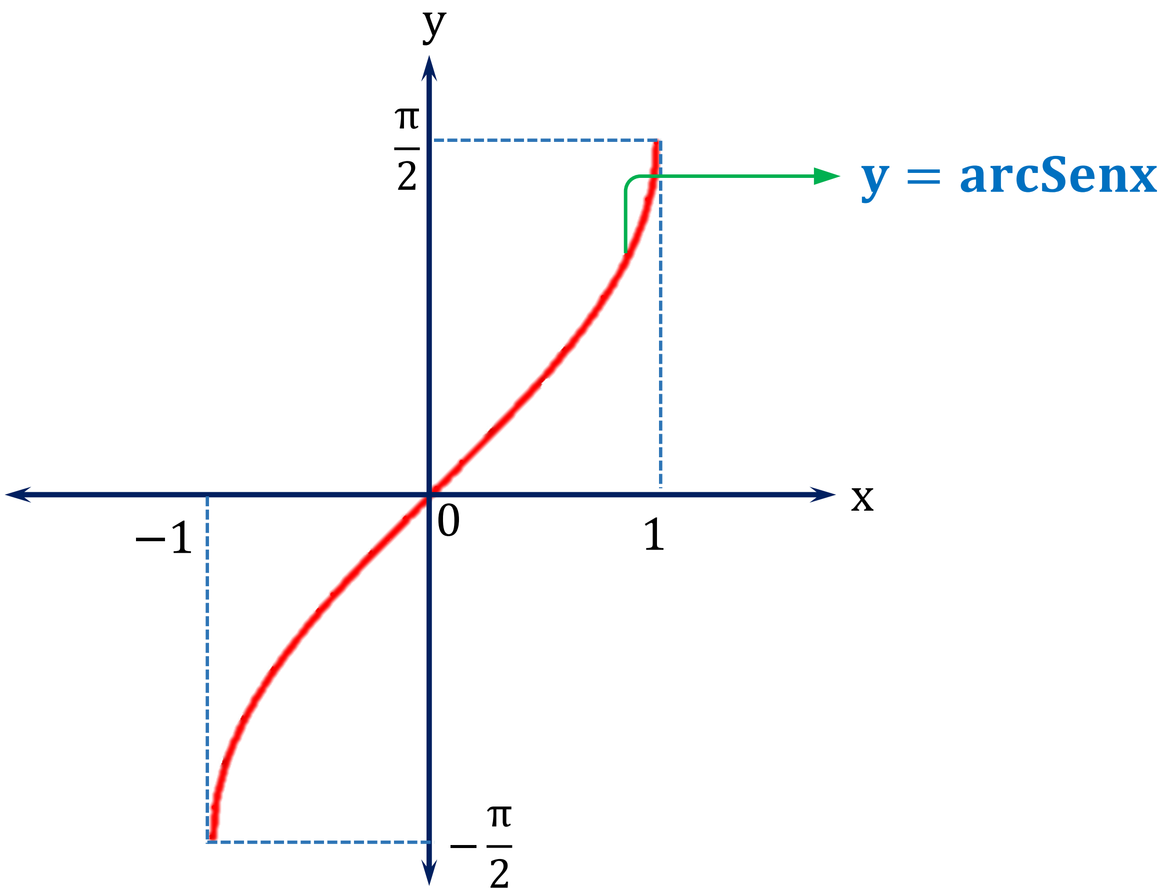
$$\therefore \theta = \frac{\pi}{4}$$

FUNCIÓN SENO:



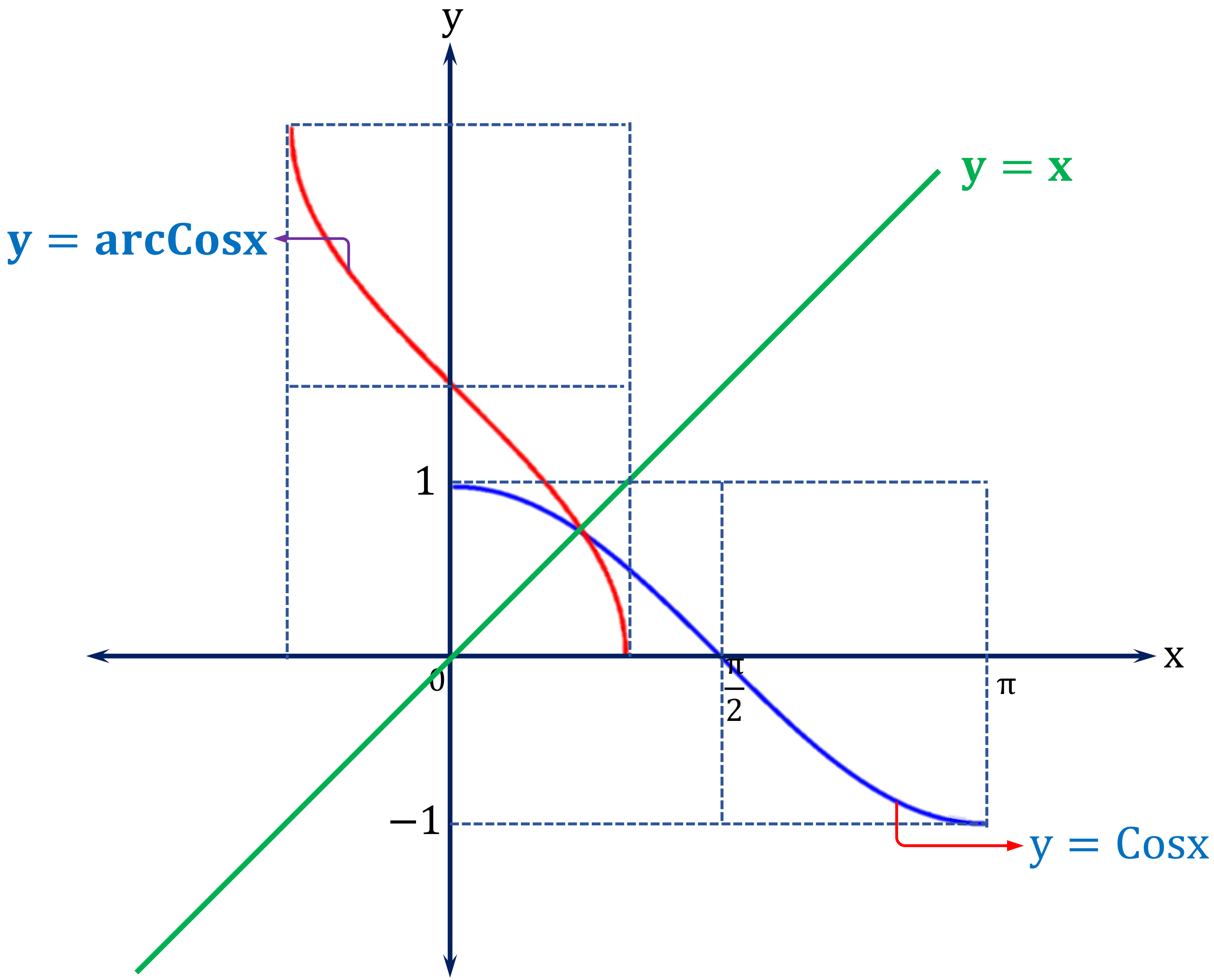
FUNCIÓN ARCO SENO:

$FT = \{(x; y) \in \mathbb{R}^2 / y = \text{arcSen}x; x \in [-1; 1]\}$



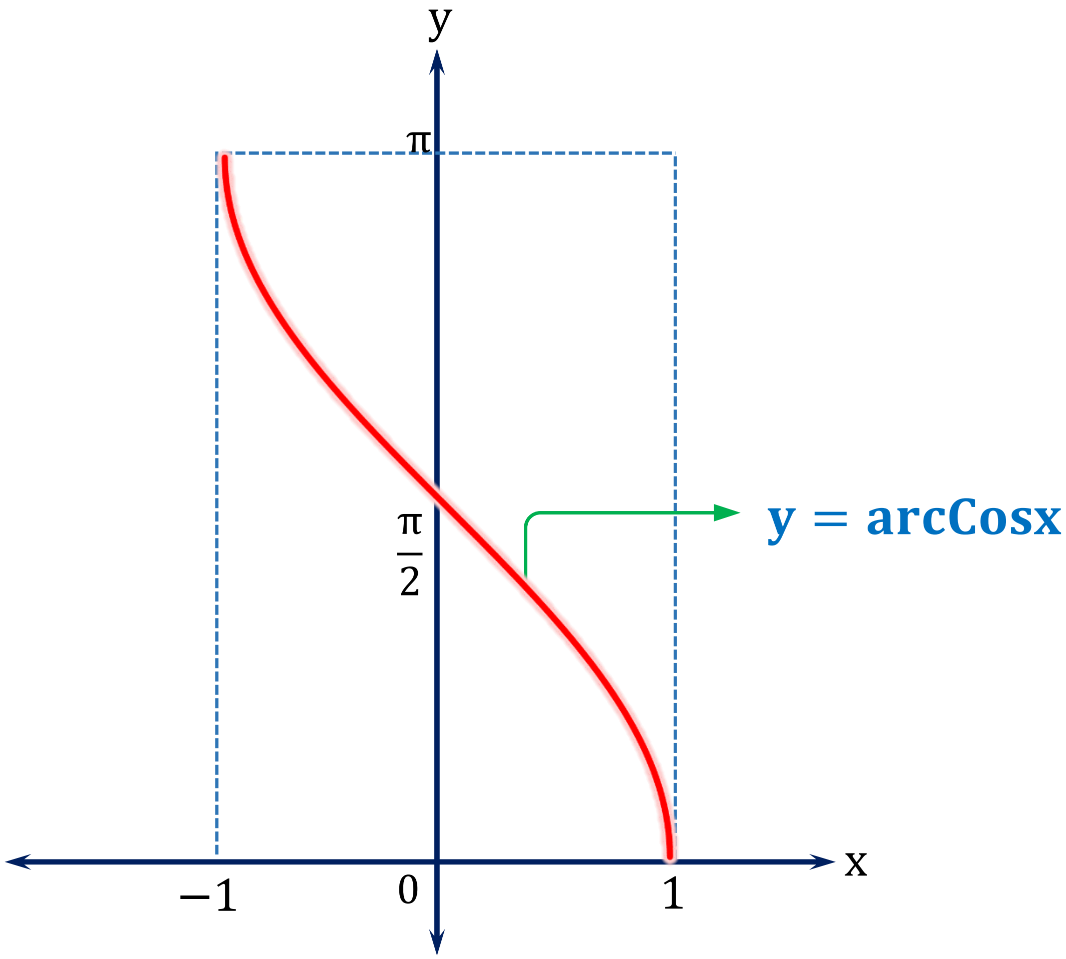
- Dominio: $D_f \in [-1; 1]$
- Rango: $R_f \in \left[-\frac{\pi}{2}; \frac{\pi}{2}\right]$
- Función: Creciente
- Función: Impar
- Función: No es periódica

FUNCIÓN COSENO:



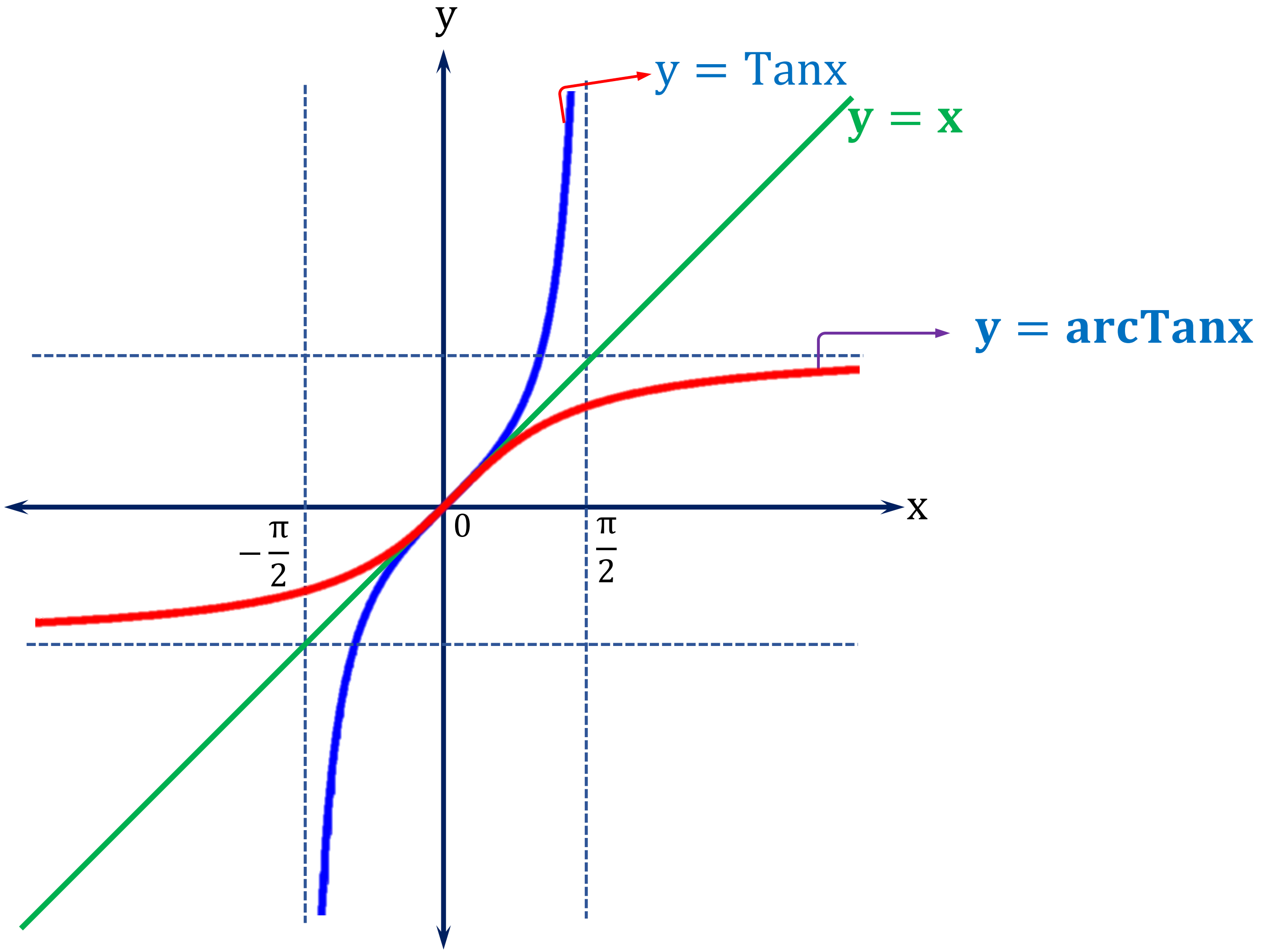
FUNCIÓN ARCO COSENO:

$$FT = \{(x;y) \in \mathbb{R}^2 / y = \text{arcCos}x; x \in [-1;1]\}$$



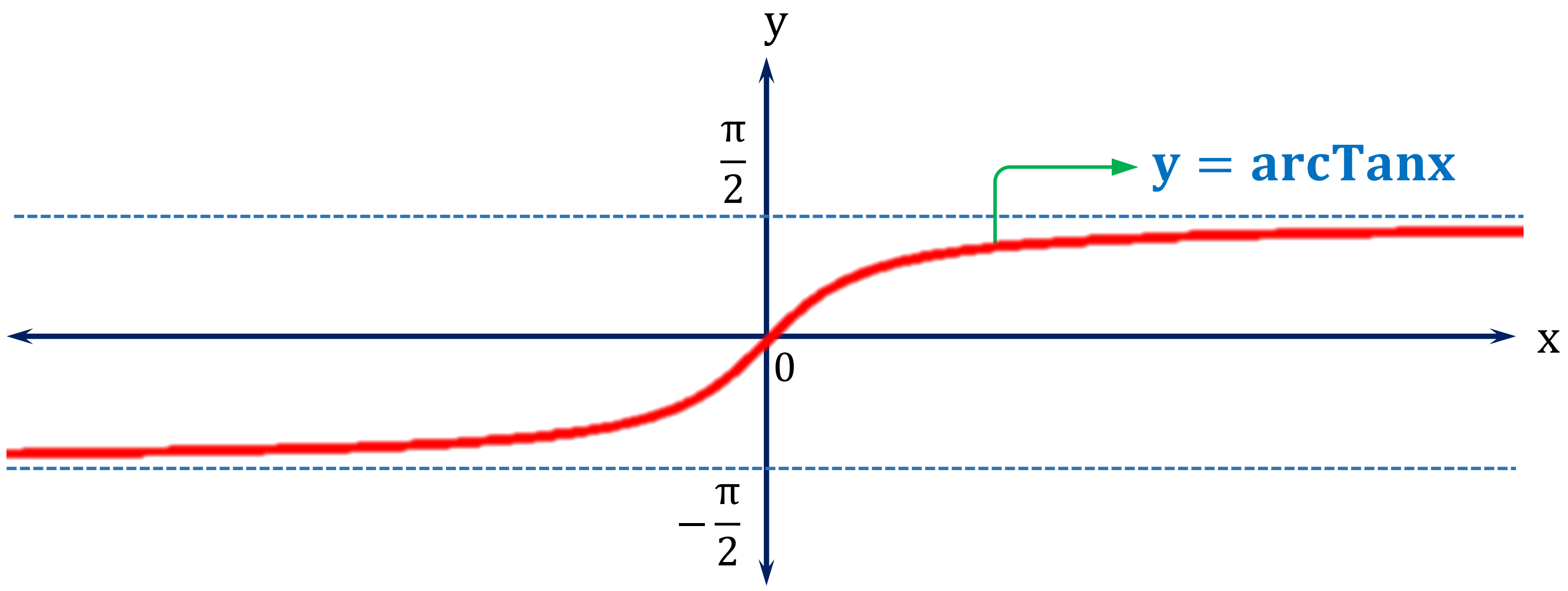
- Dominio: $D_f \in [-1; 1]$
- Rango: $R_f \in [0; \pi]$
- Función: Decreciente
- Función: No es par , ni impar
- Función: No es periódica

FUNCIÓN TANGENTE:



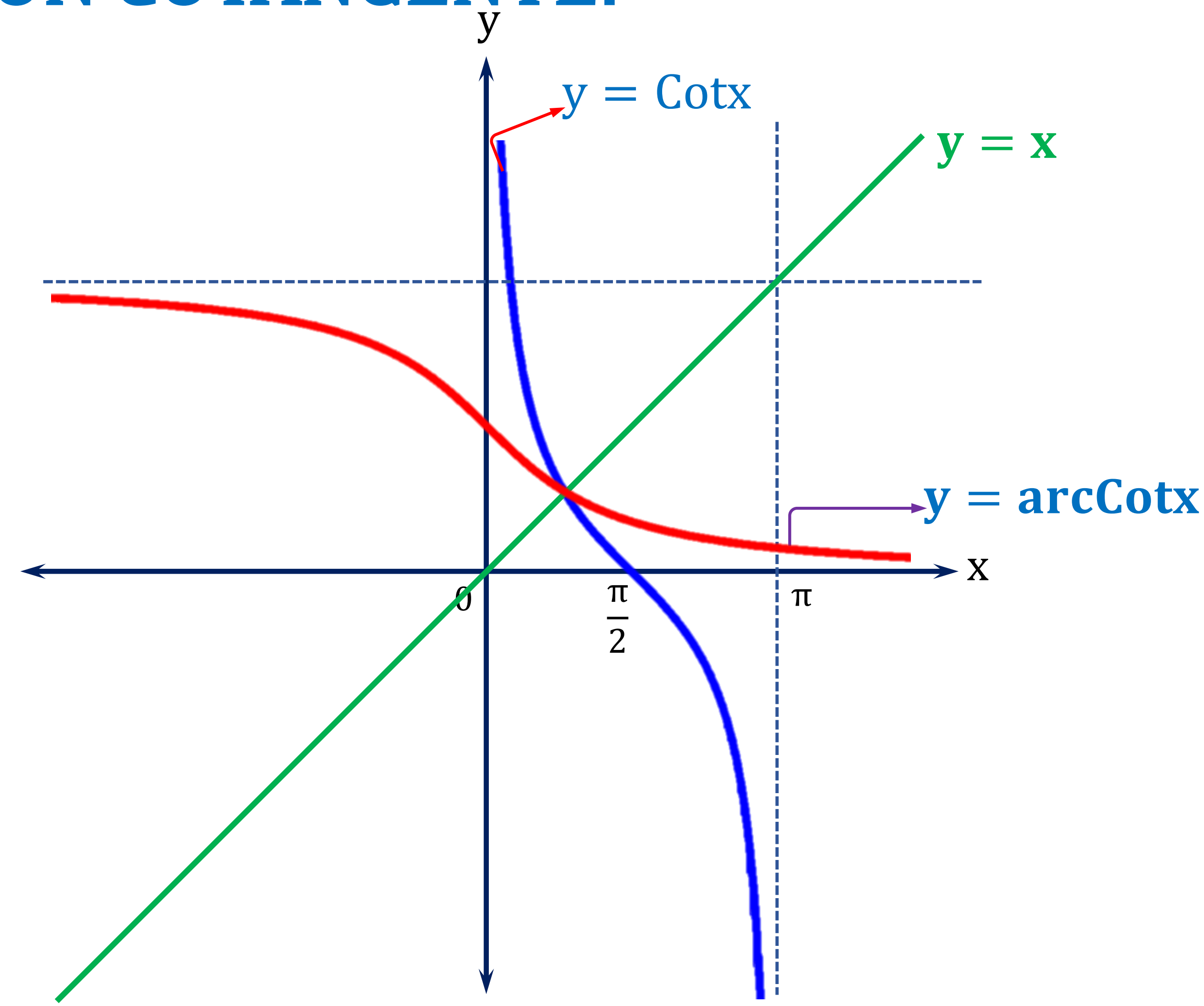
FUNCIÓN ARCO TANGENTE:

$$FT = \{(x; y) \in \mathbb{R}^2 / y = \text{arcTan}x; x \in \mathbb{R}\}$$



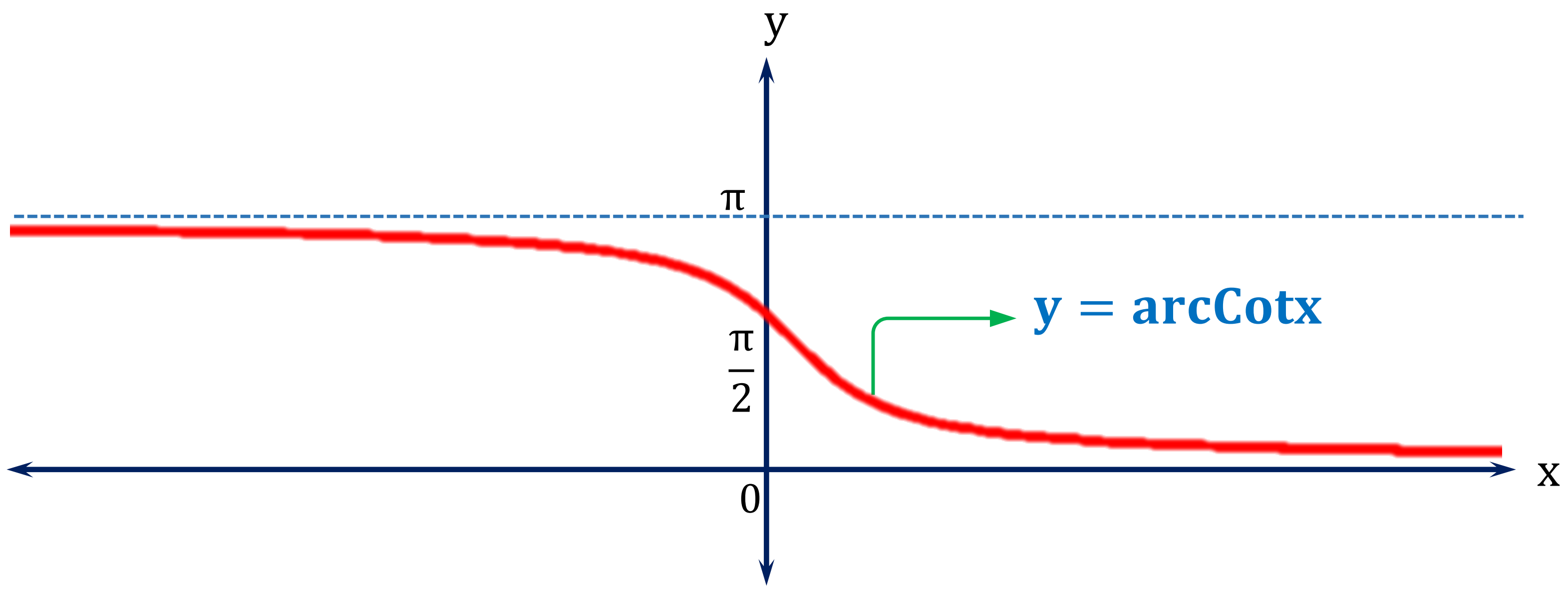
- Dominio: $D_f \in \mathbb{R}$
- Rango: $R_f \in \left]-\frac{\pi}{2}; \frac{\pi}{2}\right[$
- Función: Creciente
- Función: Impar
- Función: No es periódica

FUNCIÓN COTANGENTE:



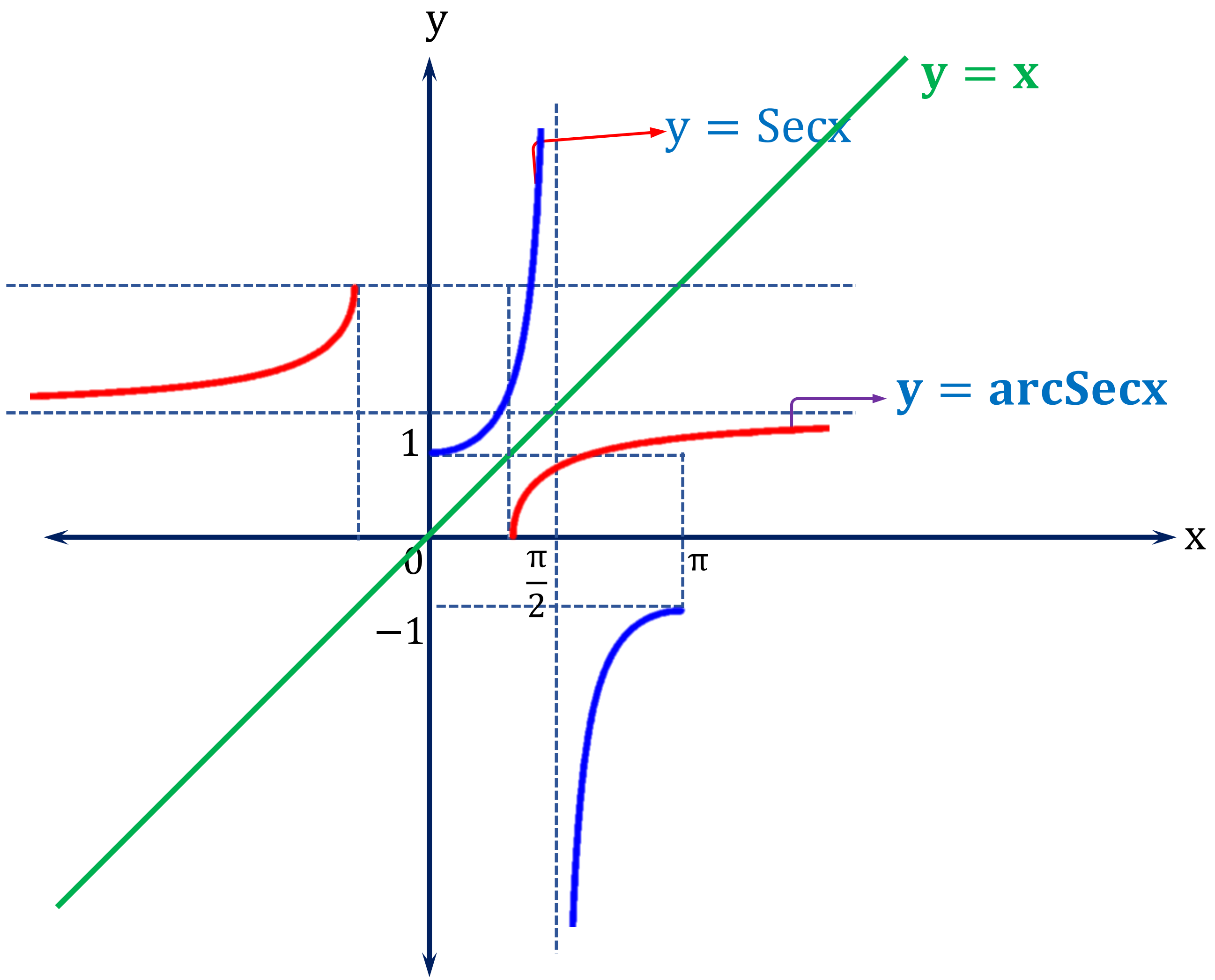
FUNCIÓN ARCO COTANGENTE:

$$FT = \{(x; y) \in \mathbb{R}^2 / y = \text{arcCot}x; x \in \mathbb{R}\}$$



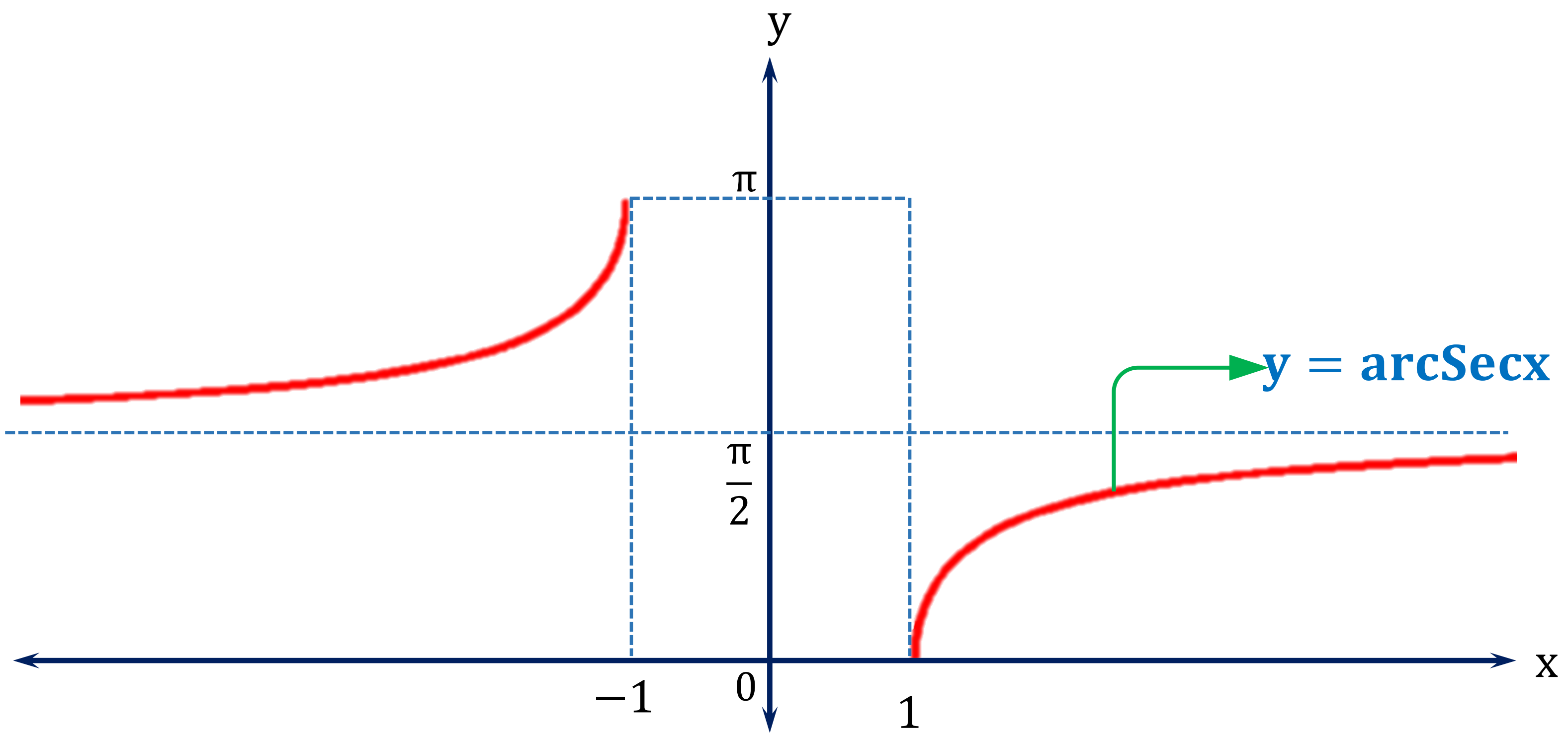
- Dominio: $D_f \in \mathbb{R}$
- Rango: $R_f \in]0; \pi[$
- Función: Decreciente
- Función: No es par, ni impar
- Función: No es periódica

FUNCIÓN SECANTE:



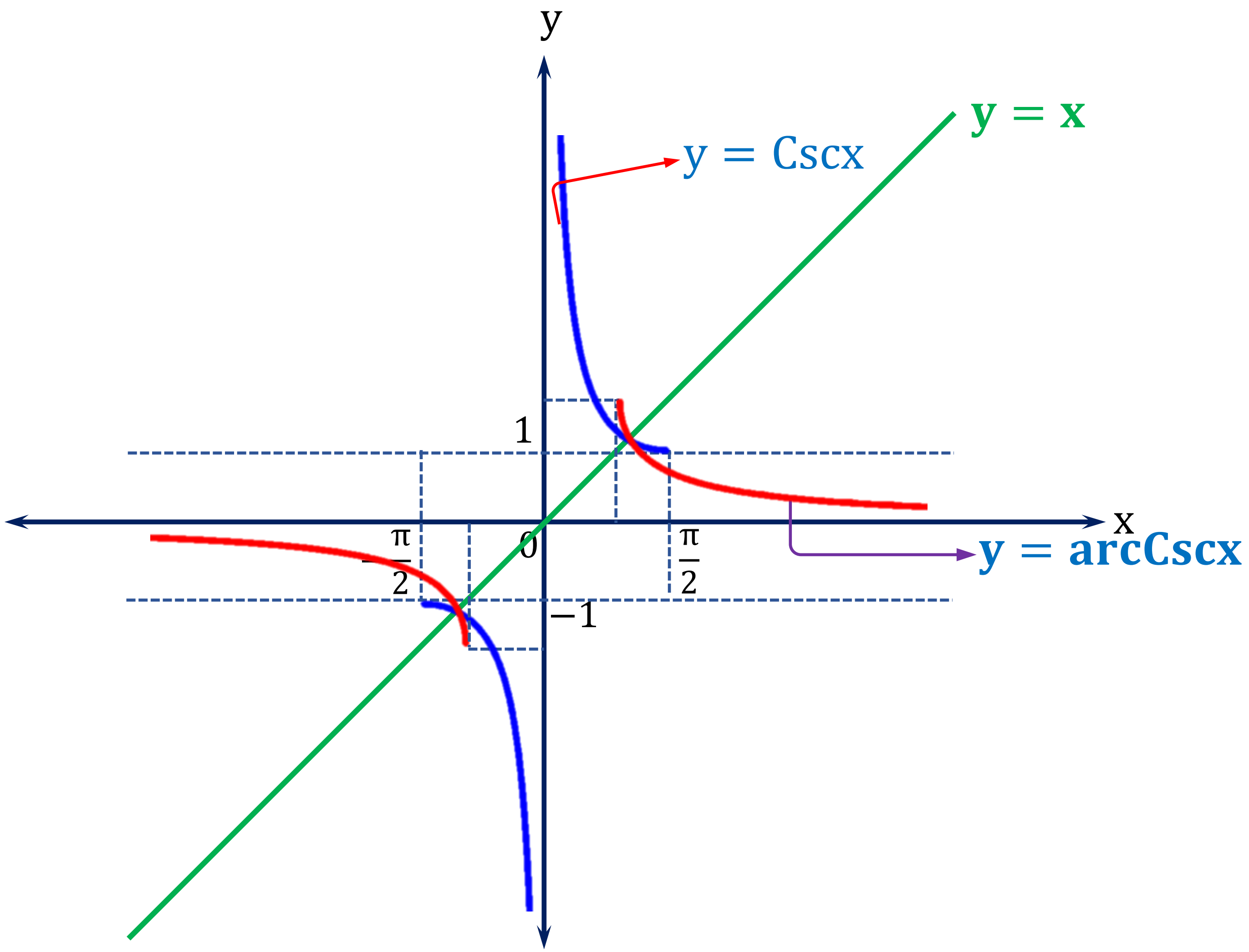
FUNCIÓN ARCO SECANTE:

$$\mathrm{FT} = \{(x;y) \in \mathbb{R}^2 / y = \mathrm{arcSec}x;\ x \in \mathbb{R} -]-1;1[\}$$



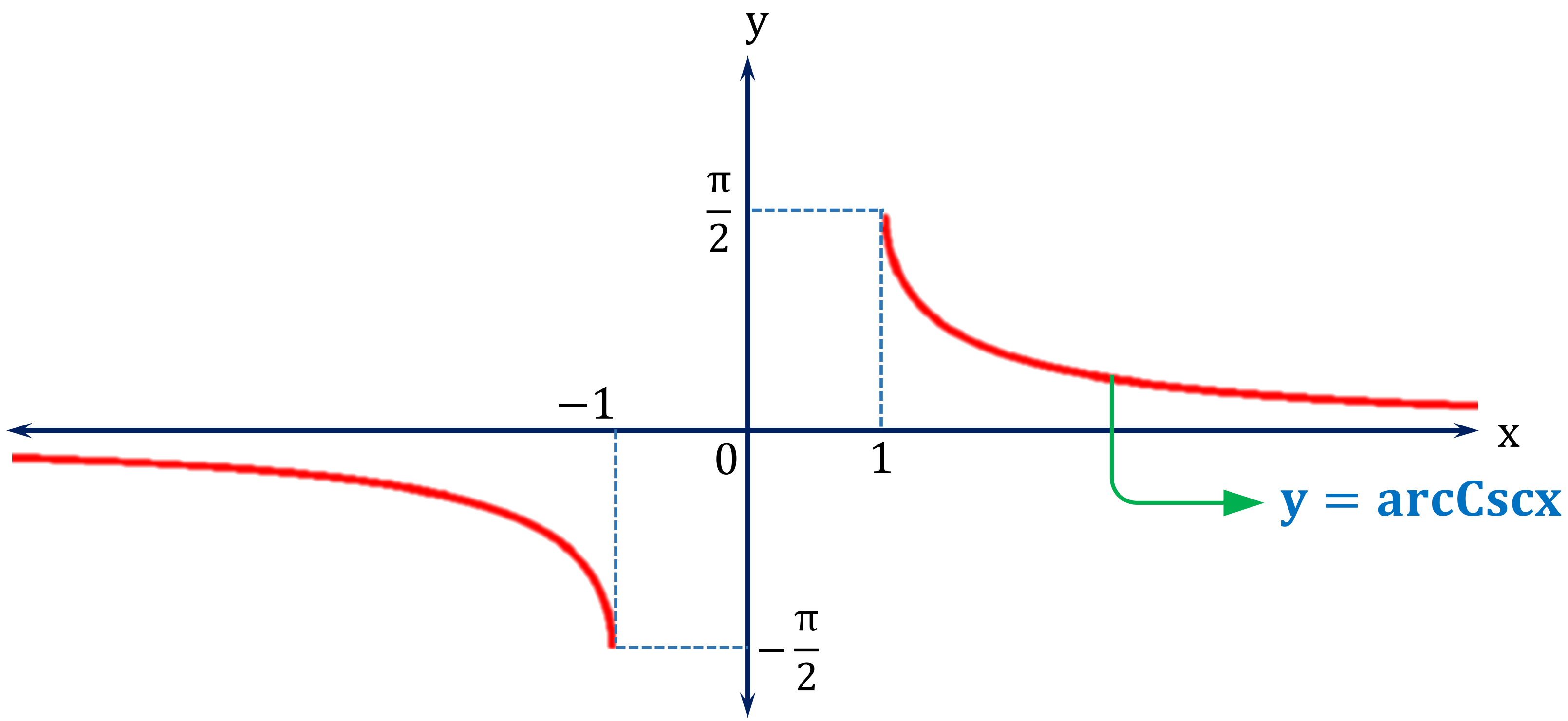
- Dominio: $D_f \in \mathbb{R} -]-1;1[$
- Rango: $R_f \in [0;\pi] - \left\{\frac{\pi}{2}\right\}$
- Función: Creciente por intervalos
- Función: No es par, ni impar
- Función: No es periódica

FUNCIÓN COSECANTE:



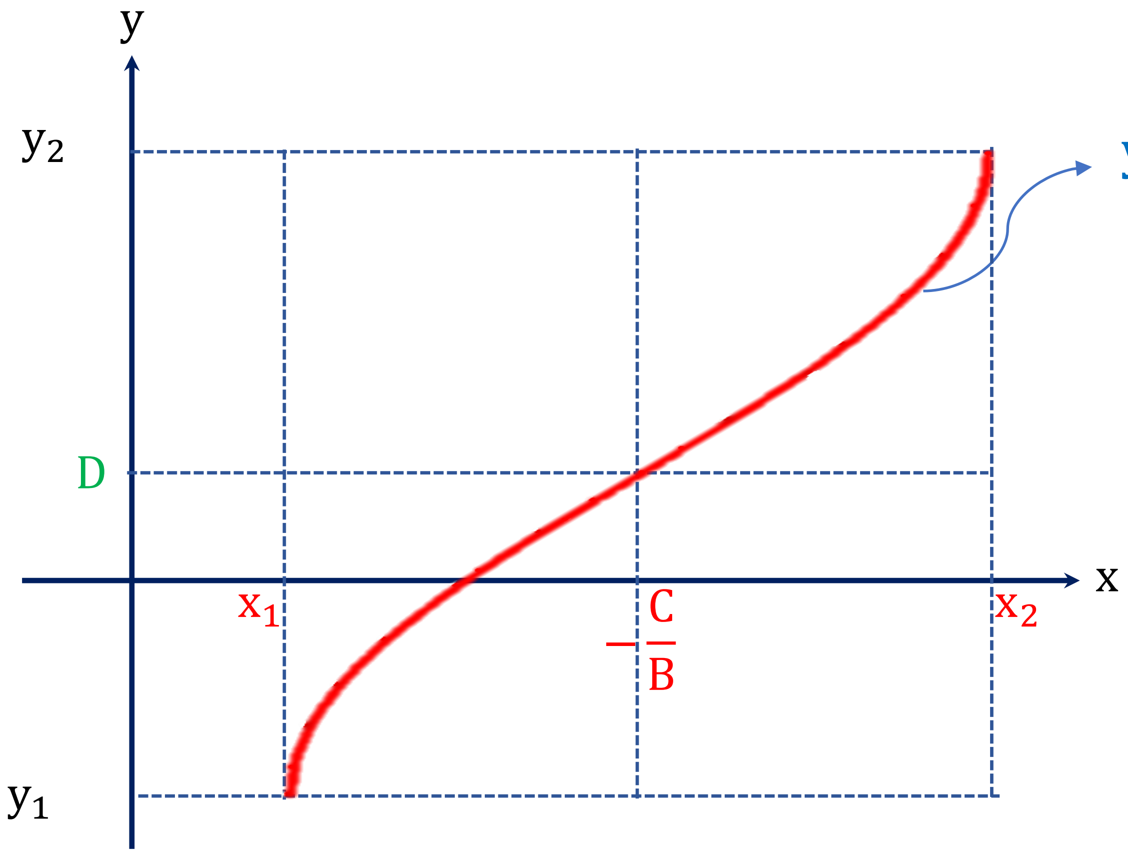
FUNCIÓN ARCO COSECANTE:

$$FT = \{(x;y) \in \mathbb{R}^2 / y = \text{arcCsc}x;\, x \in \mathbb{R} -]-1;1[\}$$



- Dominio: $D_f \in \mathbb{R} -]-1;1[$
- Rango: $R_f \in \left[-\frac{\pi}{2};\frac{\pi}{2}\right] - \{0\}$
- Función: Decreciente por intervalos
- Función: Impar
- Función: No es periódica

Gráfica Generalizada :



$\mathbf{y = A.arcSen(Bx + C) + D}$

- $A = \dfrac{y_2 - y_1}{\pi}$
- $B = \dfrac{2}{x_2 - x_1}$
- $\dfrac{-C}{B} = \dfrac{x_1 + x_2}{2}$
- $D = \dfrac{y_1 + y_2}{2}$

Observación: Arc(Cos, Cot, Sec)

$$D = \frac{y_1 + y_2}{2} - \frac{A\pi}{2}$$

MOMENTO DE PRACTICAR

PROBLEMAS Y RESOLUCIÓN

FUNCIONES TRIGONOMÉTRICAS INVERSAS

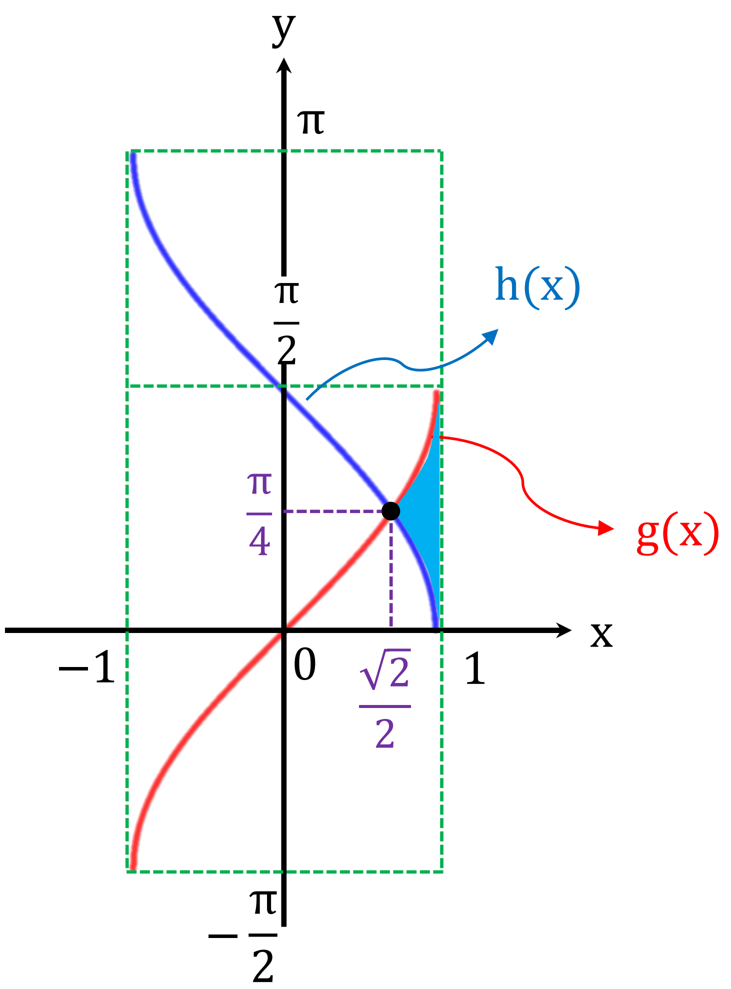
1. Determine la intersección entre el dominio y rango de la función F definida como: $F(x) = \sqrt{2(\text{arcSenx} - \text{arcCosx})}$

Resolución:

$$F(x) = \sqrt{2(\underbrace{\text{arcSenx} - \text{arcCosx}}_{\geq 0})}$$

$$\text{arcSenx} - \text{arcCosx} \geq 0$$

$$\underbrace{\text{arcSenx}}_{g(x)} \geq \underbrace{\text{arcCosx}}_{h(x)}$$



$Del\ gráfico$:

$$\mathbf{D_F}: \left[\frac{\sqrt{2}}{2}; 1\right]$$

FUNCIONES TRIGONOMÉTRICAS INVERSAS

$$F(x) = \sqrt{2(\underbrace{\text{arcSenx}}_{\frac{\pi}{2} - \text{arcCosx}} - \text{arcCosx})}$$

$$\sqrt{2\left(\frac{\pi}{2} - 2\text{arcCosx}\right)}$$

$$\sqrt{\pi - 4\text{arcCosx}}$$

Del gráfico:

$$0 \leq \text{arcCosx} \leq \frac{\pi}{4}$$

$$0 \leq 4\text{arcCosx} \leq \pi$$

$$0 \geq -4\text{arcCosx} \geq \pi$$

$$\pi \geq \pi - 4\text{arcCosx} \geq 0$$

$$\sqrt{\pi} \geq \underbrace{\sqrt{\pi - 4\text{arcCosx}}}_{F(x)} \geq 0$$

$$\mathbf{R_F: [0; \sqrt{\pi}]}$$

$$\therefore \mathbf{D_F \cap R_F: \left[\frac{\sqrt{2}}{2}; 1\right]}$$

CLAVE: C

FUNCIONES TRIGONOMÉTRICAS INVERSAS

2. Halle la suma de los valores máximo y mínimo que toma la función f definida por $f(x) = \text{arcSen}x + \text{Sen}x$

Resolución:

$$f(x) = \text{arcSen}x + \text{Sen}x$$

Analizando el dominio de F:

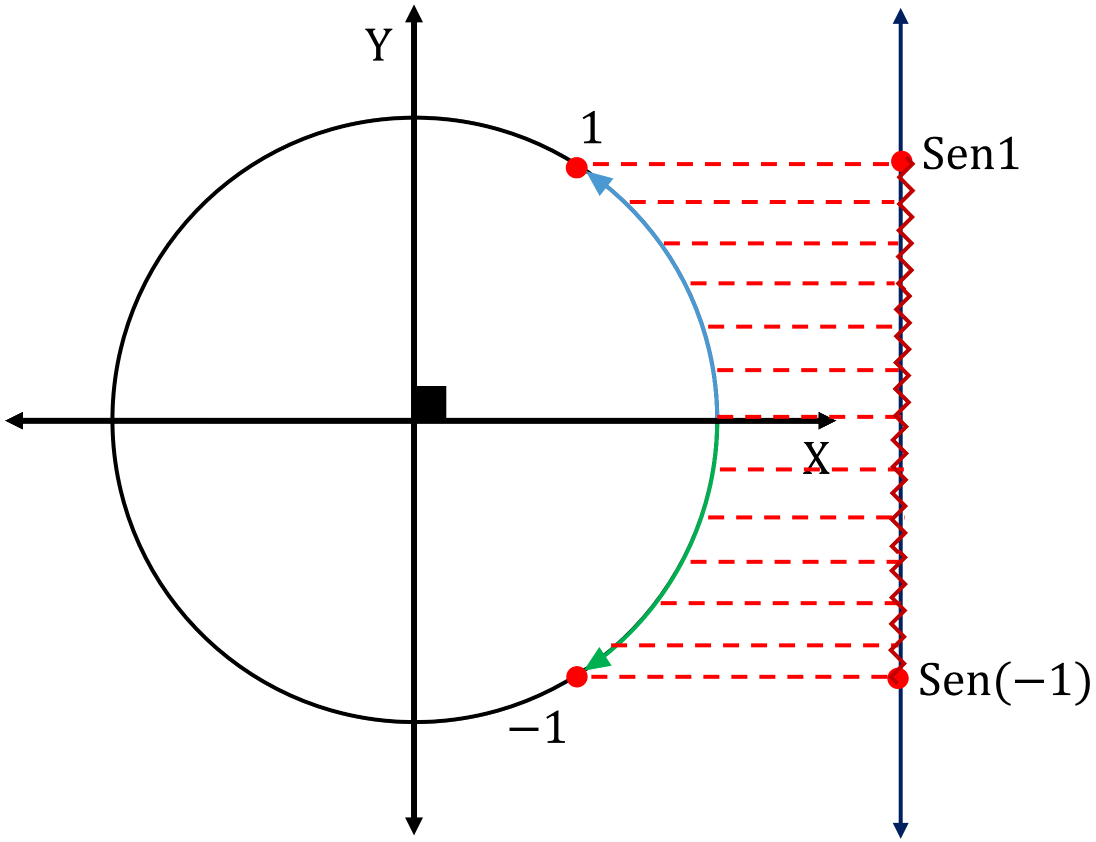
$$\underbrace{-1 \leq x \leq 1 \quad \wedge \quad x \in \mathbb{R}}_{-1 \leq x \leq 1}$$

$$-\frac{\pi}{2} \leq \text{arcSen}x \leq \frac{\pi}{2}$$

$$\text{Sen}(-1) \leq \text{Sen}x \leq \text{Sen}(1)$$

$+$

$$\underbrace{-\frac{\pi}{2} - \text{Sen}1}_{F_{Mín}} \leq \text{arcSen}x + \text{Sen}x \leq \underbrace{\frac{\pi}{2} + \text{Sen}1}_{F_{Máx}}$$



CLAVE: A

3. Calcular la suma del máximo y mínimo valor de la función G definida por: $G(x) = \left[\frac{\pi}{2} + \text{arcSen}x\right]\left[\frac{\pi}{2} - \text{arcCos}x\right]$

Resolución:

$$G(x) = \left[\frac{\pi}{2} + \text{arcSen}x\right]\underbrace{\left[\frac{\pi}{2} - \text{arcCos}x\right]}_{\text{arcSen}x}$$

$$G(x) = \left[\frac{\pi}{2}\text{arcSen}x + \text{arcSen}^2x\right]$$

$$G(x) = \underbrace{\left[\text{arcSen}^2x + \frac{\pi}{2}\text{arcSen}x + \left(\frac{\pi}{4}\right)^2\right]} - \frac{\pi^2}{16}$$

$$G(x) = \left(\text{arcSen}x + \frac{\pi}{4}\right)^2 - \frac{\pi^2}{16}$$

$$-\frac{\pi}{2} \le \text{arcSen}x \le \frac{\pi}{2}$$

$$-\frac{\pi}{4} \le \text{arcSen}x + \frac{\pi}{4} \le \frac{3\pi}{4}$$

$$0 \le \left(\text{arcSen}x + \frac{\pi}{4}\right)^2 \le \frac{9\pi^2}{16}$$

$$\underbrace{-\frac{\pi^2}{16}}_{G_{mín}} \le \left(\text{arcSen}x + \frac{\pi}{4}\right)^2 \le \underbrace{\frac{8\pi^2}{16}}_{G_{máx}}$$

$$\therefore \mathbf{G_{máx} + G_{mín} = \frac{7\pi^2}{16}}$$

CLAVE: D

4. Si la función F se define como: $F(x) = \mathrm{Sen}^{-1}\left|\frac{2x}{\pi}\right| - \mathrm{Cos}^{-1}\left|\frac{2x}{\pi}\right|$ Halle el $D_F \cap R_F$

Resolución:

$$\underbrace{F(x) = \mathrm{arcSen}\underbrace{\left|\frac{2x}{\pi}\right|}_{-1 \leq N \leq 1} - \mathrm{arcCos}\left|\frac{2x}{\pi}\right|}_{0 \leq |M|}$$

$$0 \leq \left|\frac{2x}{\pi}\right| \leq 1$$

$$-1 \leq \frac{2x}{\pi} \leq 1$$

$$-\frac{\pi}{2} \leq x \leq \frac{\pi}{2}$$

$$F(x) = \left[\frac{\pi}{2} - \mathrm{arcCos}\left|\frac{2x}{\pi}\right|\right] - \mathrm{arcCos}\left|\frac{2x}{\pi}\right|$$

$$F(x) = \frac{\pi}{2} - 2\mathrm{arcCos}\underbrace{\left|\frac{2x}{\pi}\right|}_{0 \leq N}$$

$$0 \leq \mathrm{arcCos}\left|\frac{2x}{\pi}\right| \leq \frac{\pi}{2}$$

$$-\pi \leq -2\mathrm{arcCos}\left|\frac{2x}{\pi}\right| \leq 0$$

$$-\frac{\pi}{2} \leq \underbrace{\frac{\pi}{2} - 2\mathrm{arcCos}\left|\frac{2x}{\pi}\right|}_{F(x)} \leq -\frac{\pi}{2}$$

$$\mathbf{D_F = R_F}$$

$$\therefore \mathbf{D_F \cap R_F : \left[-\frac{\pi}{2};\frac{\pi}{2}\right]}$$

CLAVE: C

5. Halle el rango de la función F, definida por: $F(x)=\text{arcSen}(\text{Sen}^4x+\text{Cos}^4x-1)$

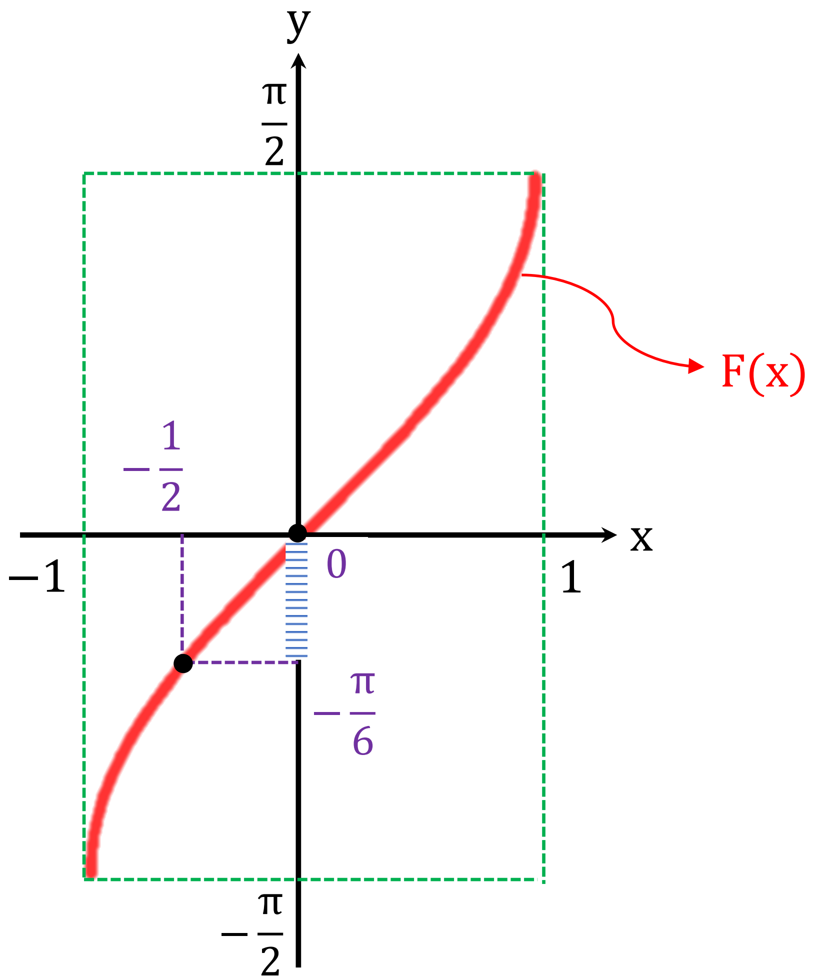
Resolución:

$$F(x)=\text{arcSen}(\text{Sen}^4x+\text{Cos}^4x-1)$$

$$\frac{1}{2^{n-1}}\le \text{Sen}^{2n}x+\text{Cos}^{2n}x\le 1,\ \forall\, x\in\mathbb{R}\ \wedge n\in\mathbb{Z}^+$$

$$\frac{1}{2^1}\le \text{Sen}^4x+\text{Cos}^4x\le 1$$

$$-\frac{1}{2}\le \underbrace{\text{Sen}^4x+\text{Cos}^4x-1}_{N}\le 0$$



$$\therefore\ \mathbf{R_F}:\left[-\frac{\pi}{6};0\right]$$

CLAVE: A

FUNCIONES TRIGONOMÉTRICAS INVERSAS

6. Sea F una función definida por: $F(x) = 3 + 5\text{arcCos}(x^2 + x + 1)$ Determine el rango de F.

Resolución:

$$F(x) = 3 + 5\text{arcCos}(\underbrace{x^2 + x + 1}_{N})$$

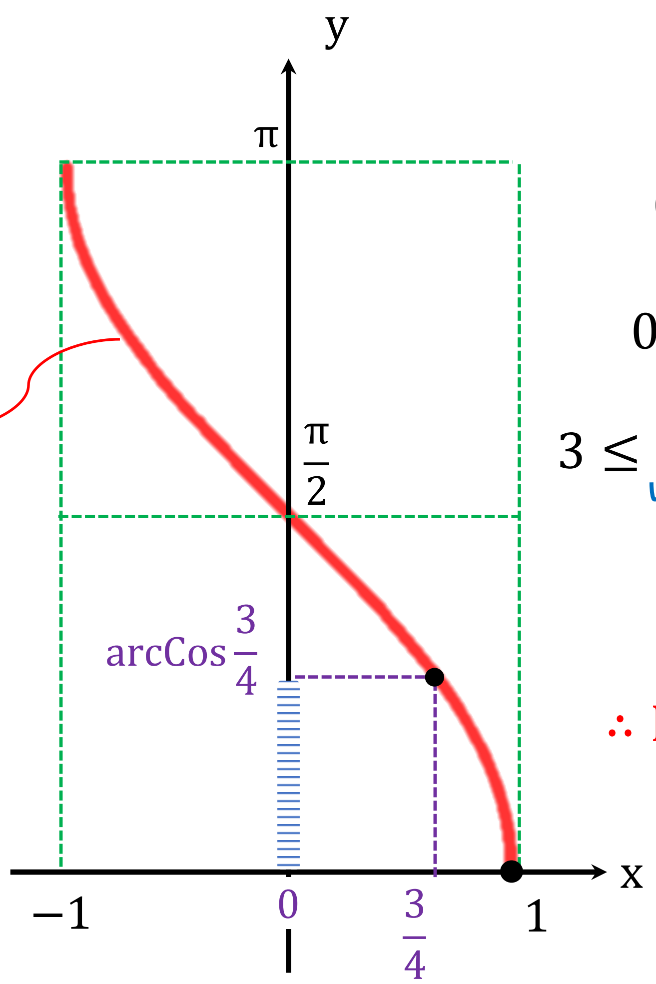
$$N = \underbrace{x^2 + 1x + \frac{1}{2^2}} - \frac{1}{4} + 1$$

$$N = \left(x + \frac{1}{2}\right)^2 + \frac{3}{4}$$

$$\left(x + \frac{1}{2}\right)^2 \geq 0$$

$$\underbrace{\left(x + \frac{1}{2}\right)^2 + \frac{3}{4}} \geq \frac{3}{4}$$

$$N \geq \frac{3}{4} \longrightarrow \frac{3}{4} \leq N \leq 1$$



$$0 \leq \text{arcCos}N \leq \text{arcCos}\frac{3}{4}$$

$$0 \leq 5\text{arcCos}N \leq 5\text{arcCos}\frac{3}{4}$$

$$3 \leq \underbrace{3 + 5\text{arcCos}N}_{F(x)} \leq 3 + 5\text{arcCos}\frac{3}{4}$$

$$\therefore R_F: \left[3; 3 + 5\text{arcCos}\frac{3}{4}\right]$$

CLAVE: C

7.Hallar x para que se cumpla que: $\text{arcCosx} - \text{arcSenx} = \text{arcCos}(x\sqrt{3})$ Hallar la suma de las soluciones.

Resolución:

$$\underbrace{\text{arcCosx}}_{\frac{\pi}{2} - \text{arcSenx}} - \text{arcSenx} = \text{arcCos}(x\sqrt{3})$$

$$\frac{\pi}{2} - 2\underbrace{\text{arcSenx}}_{\alpha} = \underbrace{\text{arcCos}(x\sqrt{3})}_{\beta}$$

$$\boxed{\text{Sen}\alpha = x} \qquad \boxed{\text{Cos}\beta = x\sqrt{3}}$$

$$\frac{\pi}{2} - 2\alpha = \beta$$

$$\text{Cos}\left(\frac{\pi}{2} - 2\alpha\right) = \text{Cos}\beta$$

$$\text{Sen}2\alpha = \text{Cos}\beta$$

$$2\text{Sen}\alpha\text{Cos}\alpha = \text{Cos}\beta$$

$$2x\text{Cos}\alpha = x\sqrt{3}$$

$$2x\text{Cos}\alpha - x\sqrt{3} = 0$$

$$x(2\text{Cos}\alpha - \sqrt{3}) = 0$$

$$x = 0 \quad \vee \quad \text{Cos}\alpha = \frac{\sqrt{3}}{2}$$

$$\text{Cos}^2\alpha = \frac{3}{4}$$

$$1 - \text{Sen}^2\alpha = \frac{3}{4}$$

$$\text{Sen}^2\alpha = \frac{1}{4}$$

$$x^2 = \frac{1}{4}$$

$$x = -\frac{1}{2} \quad \vee \quad x = \frac{1}{2}$$

$$CS = \left\{-\frac{1}{2}\ ;0\ ;\frac{1}{2}\right\}$$

$$\therefore \sum_x = 0$$

CLAVE: E

FUNCIONES TRIGONOMÉTRICAS INVERSAS

8. Calcular: $R = \text{Tan}\left\{\frac{1}{2}\text{arcTan}\left[\text{Cos}\left(2\text{Sen}^{-1}\sqrt{\frac{7}{30}}\right)\right]\right\}$

Resolución:

$$R = \text{Tan}\left\{\frac{1}{2}\text{arcTan}\left[\text{Cos}\left(\underbrace{2\text{arcSen}\sqrt{\frac{7}{30}}}_{\alpha}\right)\right]\right\}$$

$$R = \text{Tan}\left\{\frac{1}{2}\text{arcTan}[\text{Cos}(2\alpha)]\right\}$$

$$R = \text{Tan}\left\{\frac{1}{2}\underbrace{\text{arcTan}\frac{8}{15}}_{\beta}\right\}$$

$$R = \text{Tan}\left\{\frac{\beta}{2}\right\}$$

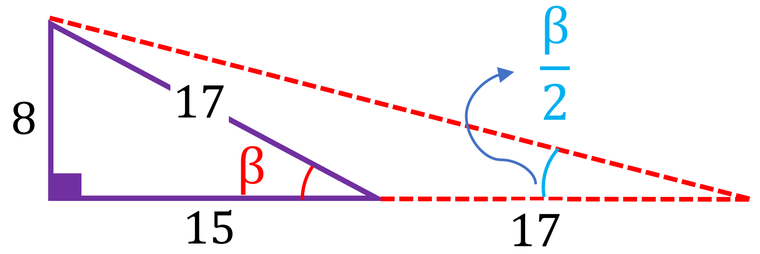
$$\therefore R = \frac{1}{4}$$

CLAVE: D

$$\boxed{\text{Sen}\alpha = \sqrt{\frac{7}{30}}}$$

$$\text{Cos}2\alpha = 1 - 2\left(\sqrt{\frac{7}{30}}\right)^2 \rightarrow \boxed{\text{Cos}2\alpha = \frac{8}{15}}$$

$$\boxed{\text{Tan}\beta = \frac{8}{15}}$$



9. Si: $x = \text{arcTan}3 + \text{arcTan}2 + \text{arcTan}1$ Halle: $\text{Tan}x$

Resolución:

$$x = \underbrace{\text{arcTan}3 + \text{arcTan}2} + \text{arcTan}1$$

$$\text{arcTan}\left(\frac{3+2}{1-3\times 2}\right) + 1\pi$$

$$x = \text{arcTan}(-1) + \pi + \text{arcTan}1$$

$$x = \cancel{-\text{arcTan}1} + \pi + \cancel{\text{arcTan}1}$$

$$x = \pi$$

$$\text{Tan}x = \text{Tan}\pi$$

$$\therefore \mathbf{Tanx = 0}$$

CLAVE: A

10. Reducir la siguiente expresión: $A=\dfrac{\sqrt{\sqrt{\operatorname{arcCos}x-\pi}+2\pi^2+2\pi\operatorname{arcSen}x}}{\operatorname{arcTan}(-x)+\operatorname{arcCos}(x+2)}$

Resolución:

$$A=\frac{\sqrt{\sqrt{\operatorname{arcCos}x-\pi}+2\pi^2+2\pi\operatorname{arcSen}x}}{\operatorname{arcTan}(-x)+\operatorname{arcCos}(x+2)}$$

$$\operatorname{arcCos}x-\pi\geq 0$$

$$\operatorname{arcCos}x\geq \pi$$

$$\operatorname{arcCos}x>\pi \quad \vee \quad \boxed{\operatorname{arcCos}x=\pi}$$

$$\rightarrow \mathbf{x=-1}$$

$$A=\frac{\sqrt{\sqrt{\pi-\pi}+2\pi^2+2\pi\operatorname{arcSen}(-1)}}{\operatorname{arcTan}(-(-1))+\operatorname{arcCos}(-1+2)}$$

$$A=\frac{\sqrt{2\pi^2+2\pi\left(-\frac{\pi}{2}\right)}}{\frac{\pi}{4}+0}$$

$$A=\frac{\sqrt{\pi^2}}{\frac{\pi}{4}}$$

$$A=\frac{\pi}{\frac{\pi}{4}}$$

$$\therefore \mathbf{A=4}$$

CLAVE: D

FUNCIONES TRIGONOMÉTRICAS INVERSAS

11. Dada la función F, definida por: $F(x) = |\text{arcSen}x| + |\text{arcTan}x|$ Halle el rango de F.

Resolución:

$$F(x) = |\text{arcSen}x| + |\text{arcTan}x|$$

Analizando el dominio de F:

$$\underbrace{-1 \leq x \leq 1 \quad \wedge \quad x \in \mathbb{R}}$$

$$-1 \leq x \leq 1$$

G

$$-\frac{\pi}{2} \leq \text{arcSen}x \leq \frac{\pi}{2} \qquad -\frac{\pi}{4} \leq \text{arcTan}x \leq \frac{\pi}{4}$$

I

$$\underbrace{0 \leq |\text{arcSen}x| \leq \frac{\pi}{2} \qquad 0 \leq |\text{arcTan}x| \leq \frac{\pi}{4}}_{\text{Crecientes}}$$

$$0 \leq |\text{arcSen}x| \leq \frac{\pi}{2}$$

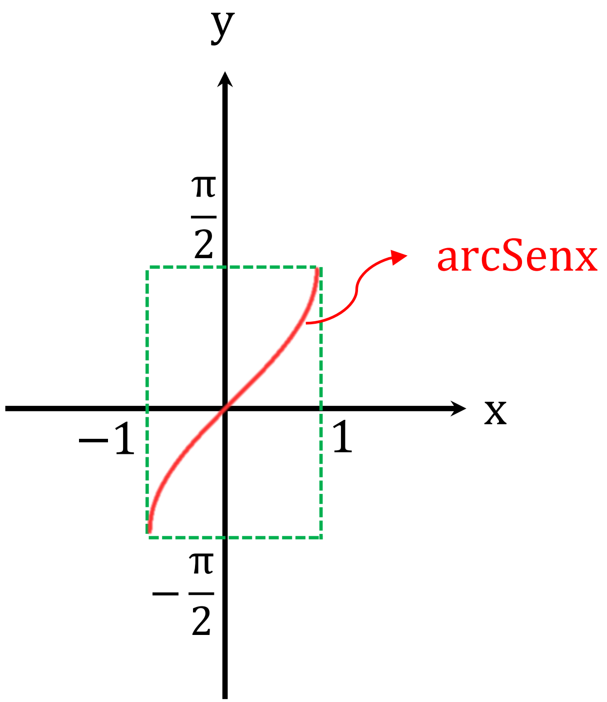
$$0 \leq |\text{arcTan}x| \leq \frac{\pi}{4}$$

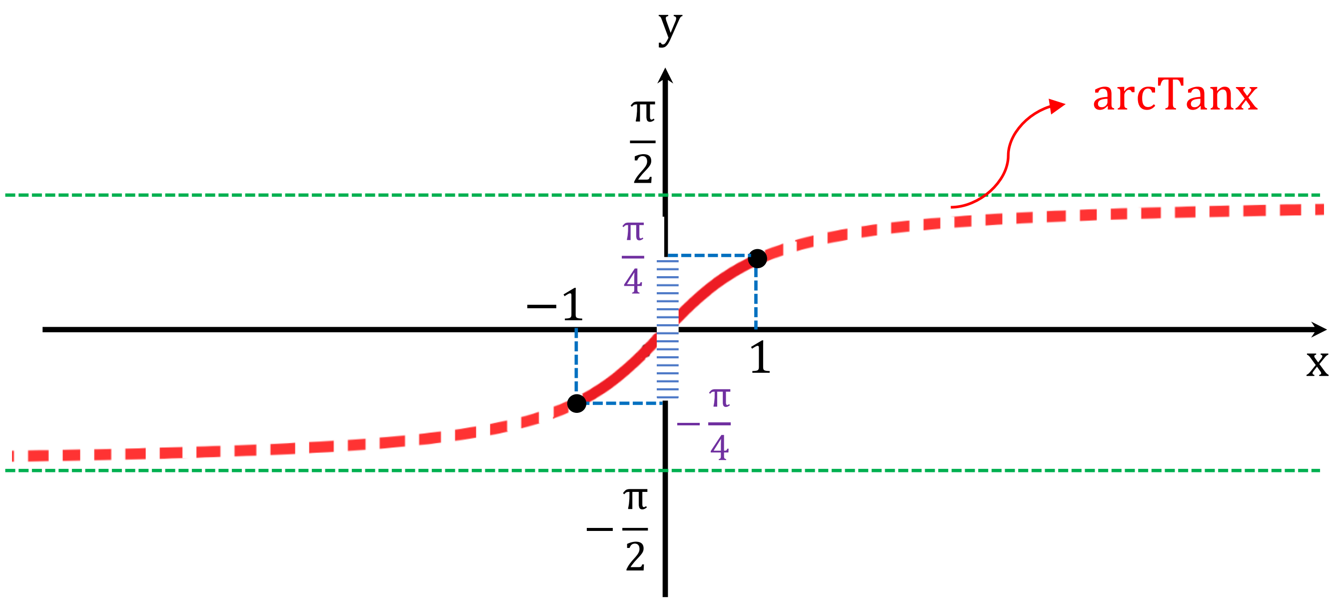
$$0 \leq \underbrace{|\text{arcSen}x| + |\text{arcTan}x|}_{F(x)} \leq \frac{3\pi}{4}$$

$$\therefore R_F: \left[0; \frac{3\pi}{4}\right]$$

CLAVE: C

FUNCIONES TRIGONOMÉTRICAS INVERSAS





H

J

12.Dada la función F, definida por: $F(x) = \text{arcTan}\left(\dfrac{|x|}{1+x^2}\right)$ Hallar el rango de F.

Resolución:

$$F(x) = \text{arcTan}\left(\underbrace{\dfrac{|x|}{1+|x|^2}}_{N}\right)$$

$$0 \le \underbrace{\text{arcTanN}}_{F(x)} \le \text{arcTan}\dfrac{1}{2}$$

$$\therefore\ R_F: \left[0; \text{arcTan}\dfrac{1}{2}\right]$$

$$N = \dfrac{|x|}{1+|x|^2}$$

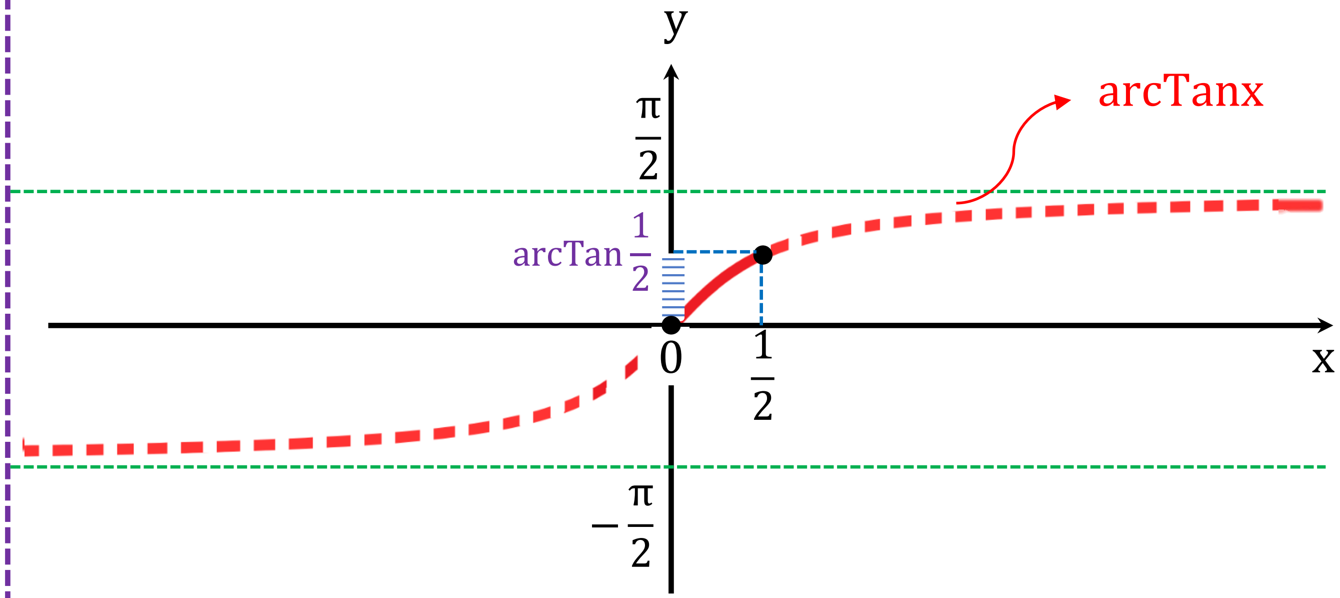
$$N = \dfrac{1}{\dfrac{1+|x|^2}{|x|}}$$

$$N = \dfrac{1}{\dfrac{1}{|x|}+|x|}$$

$$\dfrac{1}{|x|}+|x| \ge 2$$

$$\dfrac{1}{\dfrac{1}{|x|}+|x|} \le \dfrac{1}{2}$$

$$0 \le N \le \dfrac{1}{2}$$



CLAVE: A

13.Si: $\operatorname{arcTan}(\operatorname{Sec}\theta) + \operatorname{arcTan}(\operatorname{Csc}\theta) + \operatorname{arcTan}(\operatorname{Sec}\theta + \operatorname{Csc}\theta) = \pi$ Calcule: $\operatorname{arcTan}(\operatorname{Sen}2\theta)$

Resolución:

$$\underbrace{\operatorname{arcTan}(\operatorname{Sec}\theta)}_{x} + \underbrace{\operatorname{arcTan}(\operatorname{Csc}\theta)}_{y} + \underbrace{\operatorname{arcTan}(\operatorname{Sec}\theta + \operatorname{Csc}\theta)}_{z} = \pi \longrightarrow x + y + z = \pi$$

$$\operatorname{Tan}x = \operatorname{Sec}\theta \qquad \operatorname{Tan}y = \operatorname{Csc}\theta \qquad \operatorname{Tan}z = \operatorname{Sec}\theta + \operatorname{Csc}\theta$$

$$\operatorname{Tan}x + \operatorname{Tan}y + \operatorname{Tan}z = \operatorname{Tan}x\operatorname{Tan}y\operatorname{Tan}z$$

$$\operatorname{Sec}\theta + \operatorname{Csc}\theta + \operatorname{Sec}\theta + \operatorname{Csc}\theta = \operatorname{Sec}\theta.\operatorname{Csc}\theta(\operatorname{Sec}\theta + \operatorname{Csc}\theta)$$

$$2\cancel{(\operatorname{Sec}\theta + \operatorname{Csc}\theta)} = \operatorname{Sec}\theta.\operatorname{Csc}\theta\cancel{(\operatorname{Sec}\theta + \operatorname{Csc}\theta)}$$

$$2 = \frac{1}{\operatorname{Sen}\theta} \times \frac{1}{\operatorname{Cos}\theta}$$

$$\operatorname{Sen}2\theta = 1$$

$$\operatorname{arcTan}(\operatorname{Sen}2\theta) = \operatorname{arcTan}1$$

$$\therefore \mathbf{arcTan(Sen2\theta)} = \frac{\pi}{4}$$

CLAVE: C

14. Hallar la suma de los "n" primeros términos de: $S = \mathrm{Cot}^{-1}2 + \mathrm{Cot}^{-1}8 + \mathrm{Cot}^{-1}18 + \mathrm{Cot}^{-1}32 + \cdots$

Resolución:

$$S = \mathrm{arcCot}2.1^2 + \mathrm{arcCot}2.2^2 + \mathrm{arcCot}2.3^2 + \mathrm{arcCot}2.4^2 + \cdots + \mathrm{arcCot}2.n^2$$

$$S = \mathrm{arcTan}\frac{1}{2} + \mathrm{arcTan}\frac{1}{8} + \mathrm{arcTan}\frac{1}{18} + \mathrm{arcTan}\frac{1}{32} + \cdots + \mathrm{arcTan}\frac{1}{2n^2}$$

$$S = \mathrm{arcTan}\frac{2}{4} + \mathrm{arcTan}\frac{2}{16} + \mathrm{arcTan}\frac{2}{36} + \mathrm{arcTan}\frac{2}{64} + \cdots + \mathrm{arcTan}\frac{2}{4n^2}$$

$$S = \mathrm{arcTan}\frac{3-1}{1+3.1} + \mathrm{arcTan}\frac{5-3}{1+5.3} + \mathrm{arcTan}\frac{7-5}{1+7.5} + \mathrm{arcTan}\frac{9-7}{1+9.7} + \cdots + \mathrm{arcTan}\frac{(2n+1)-(2n-1)}{1+(2n+1)(2n-1)}$$

$$\boxed{\mathbf{arcTanA - arcTanB = arcTan\left(\frac{A-B}{1+A\times B}\right)}}$$

$$\cancel{\text{arcTan}3} - \text{arcTan}1$$
$$\cancel{\text{arcTan}5} - \cancel{\text{arcTan}3}$$
$$\cancel{\text{arcTan}7} - \cancel{\text{arcTan}5}$$
$$\cancel{\text{arcTan}9} - \cancel{\text{arcTan}7}$$
$$\vdots \qquad \vdots$$
$$\text{arcTan}(2n+1) - \cancel{\text{arcTan}(2n-1)} \qquad +$$

$$S = \text{arcTan}(2n+1) - \text{arcTan}1$$

$$S = \text{arcTan}\left(\frac{(2n+1)-1}{1+(2n+1)(1)}\right)$$

$$S = \text{arcTan}\left(\frac{\cancel{2}n}{\cancel{2}(n+1)}\right)$$

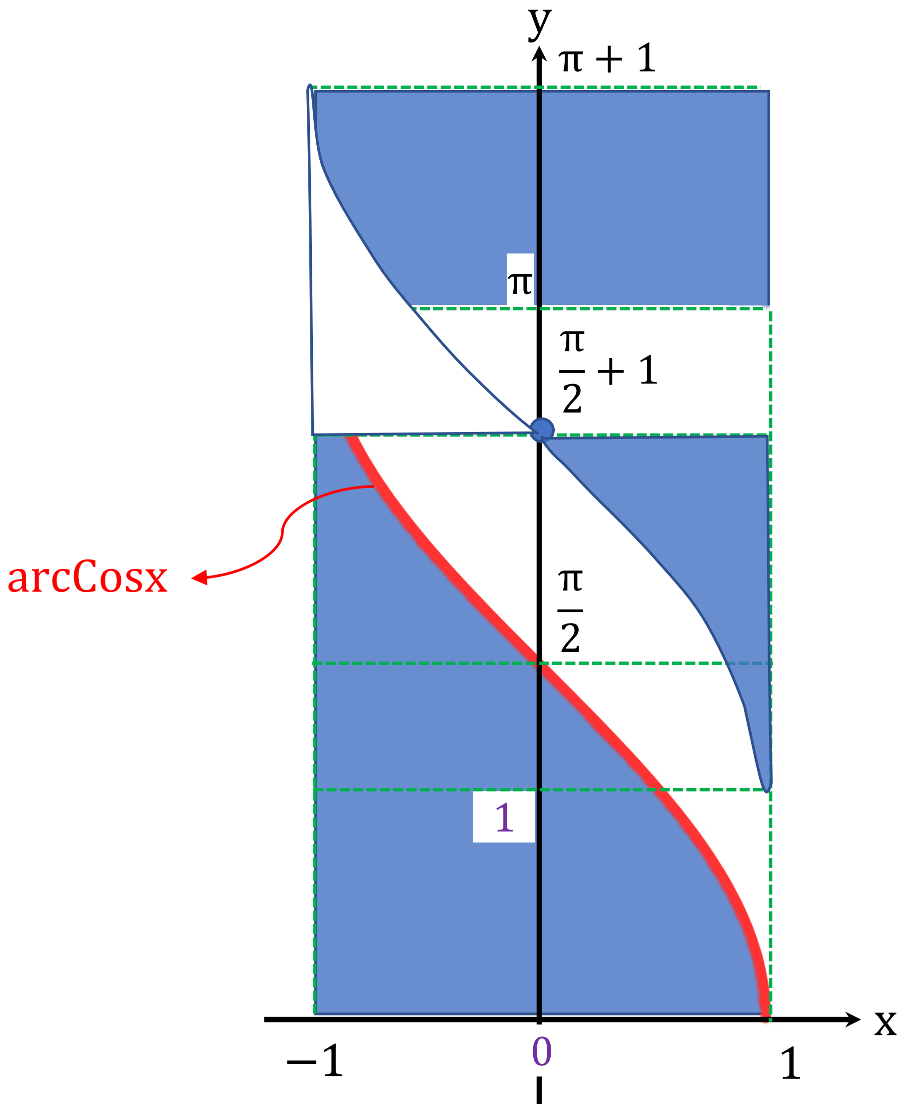
$$\therefore \mathbf{S = arcTan\left(\frac{n}{n+1}\right)}$$

CLAVE: D

15. Determinar el área de la región definida por la gráfica de la función: $F(x) = 1 + \text{arcCos}x$ Con el eje x

Resolución:

$$F(x) = 1 + \text{arcCos}x$$



$$S_x = 2\left(\frac{\pi}{2} + 1\right)$$

$$\therefore \mathbf{S_x = 2 + \pi}$$

CLAVE: D

16. Si $(x_0; y_0)$ es punto de intersección de las gráficas de las funciones F y G definidas por:

$$F(x) = \frac{3\pi}{2} - \text{arcCos}x,\ G(x) = \pi + \text{arcTan}x$$

Determine el valor de la expresión: $E = y_0 \text{Cos}\left(\frac{x_0}{\pi}\right)$

Resolución:

$$y = \frac{3\pi}{2} - \text{arcCos}x = \pi + \text{arcTan}x$$

$$\frac{\pi}{2} - \text{arcCos}x = \text{arcTan}x$$

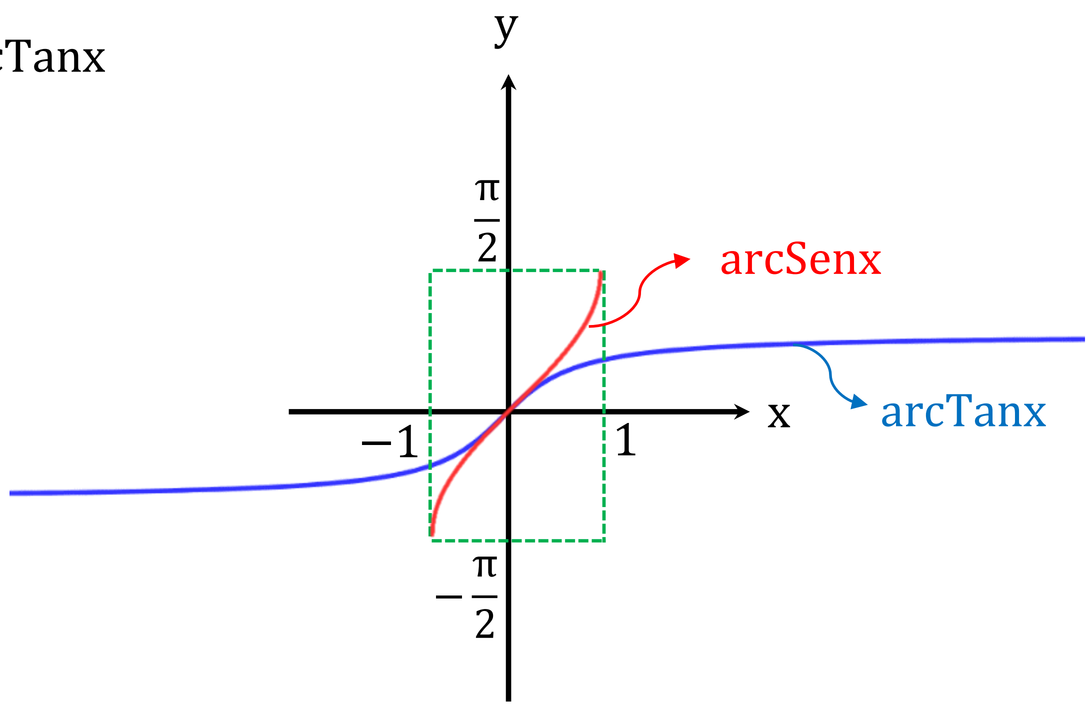
$$\text{arcSen}x = \text{arcTan}x$$

$$x_0 = 0$$

$$y_0 = \pi + \text{arcTan}x_0$$

$$y_0 = \pi + \text{arcTan}0$$

$$y_0 = \pi$$



$$E = y_0 \text{Cos}\left(\frac{x_0}{\pi}\right) \longrightarrow E = \pi\text{Cos}\left(\frac{0}{\pi}\right)$$

$$\therefore \mathbf{E = \pi}$$

CLAVE: C

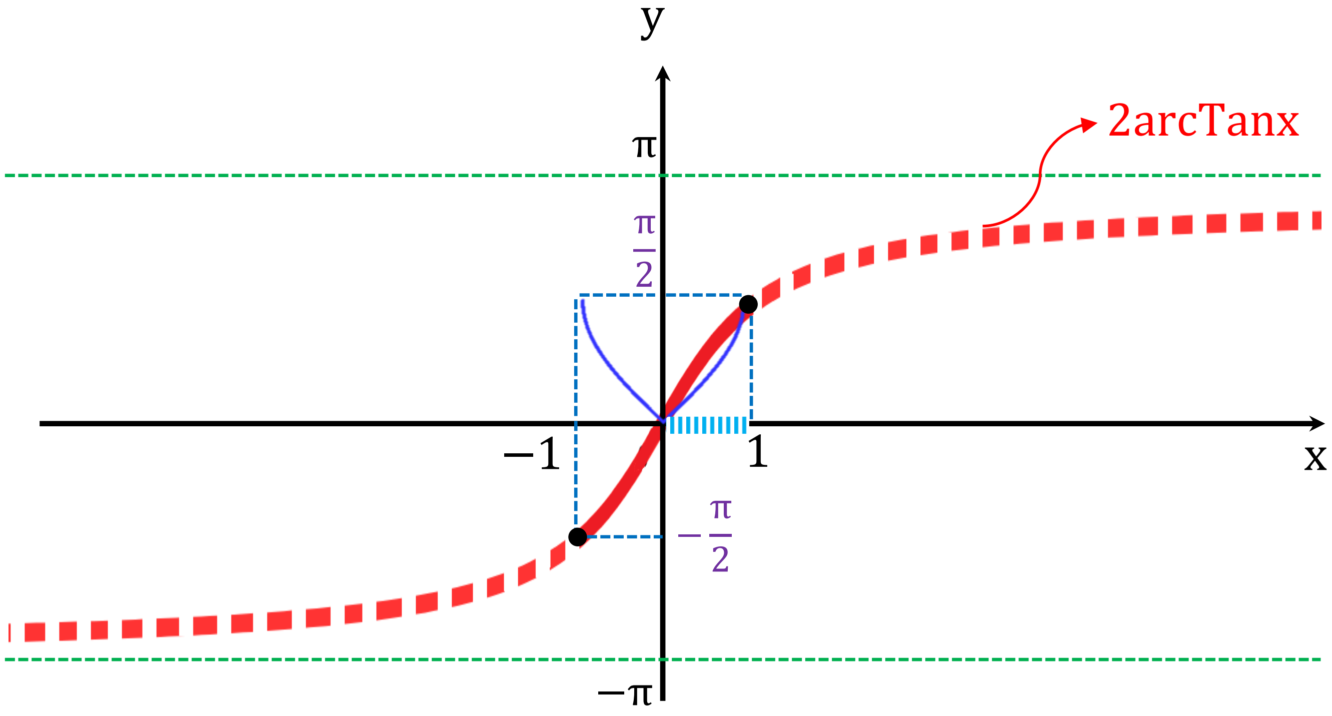
17. Resolver: $2\text{arcTan}x - |\text{arcSen}x| > 0$

Resolución:

$$|\text{arcSen}x| < 2\text{arcTan}x$$

Analizando el dominio de F:

$$\underbrace{-1 \leq x \leq 1 \quad \wedge \quad x \in \mathbb{R}}_{-1 \leq x \leq 1}$$

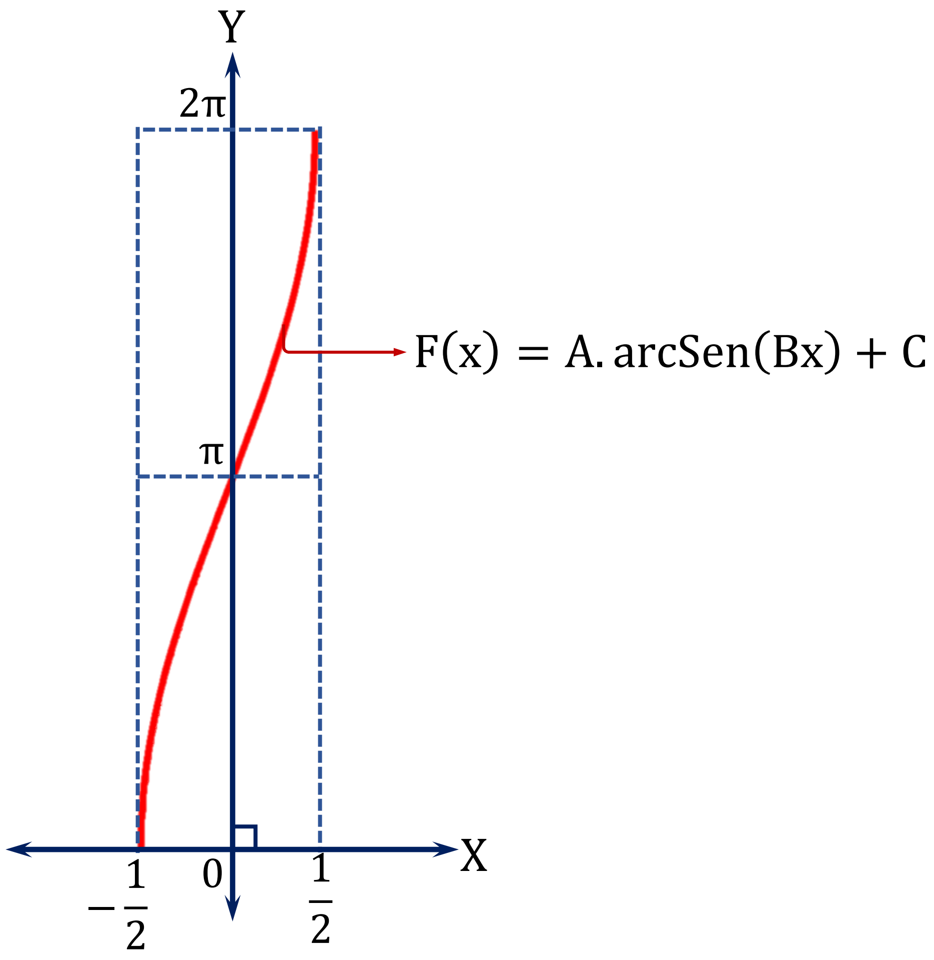


$$0 < x < 1$$

$\therefore$ **CS: $x \in \,]0; 1[$**

CLAVE: C

18. Determinar la regla de correspondencia de la función F, dada por la gráfica.



Resolución:

$$A = \frac{2\pi - 0}{\pi} \longrightarrow A = 2$$

$$B = \frac{2}{\frac{1}{2} - \left(-\frac{1}{2}\right)} \longrightarrow B = 2$$

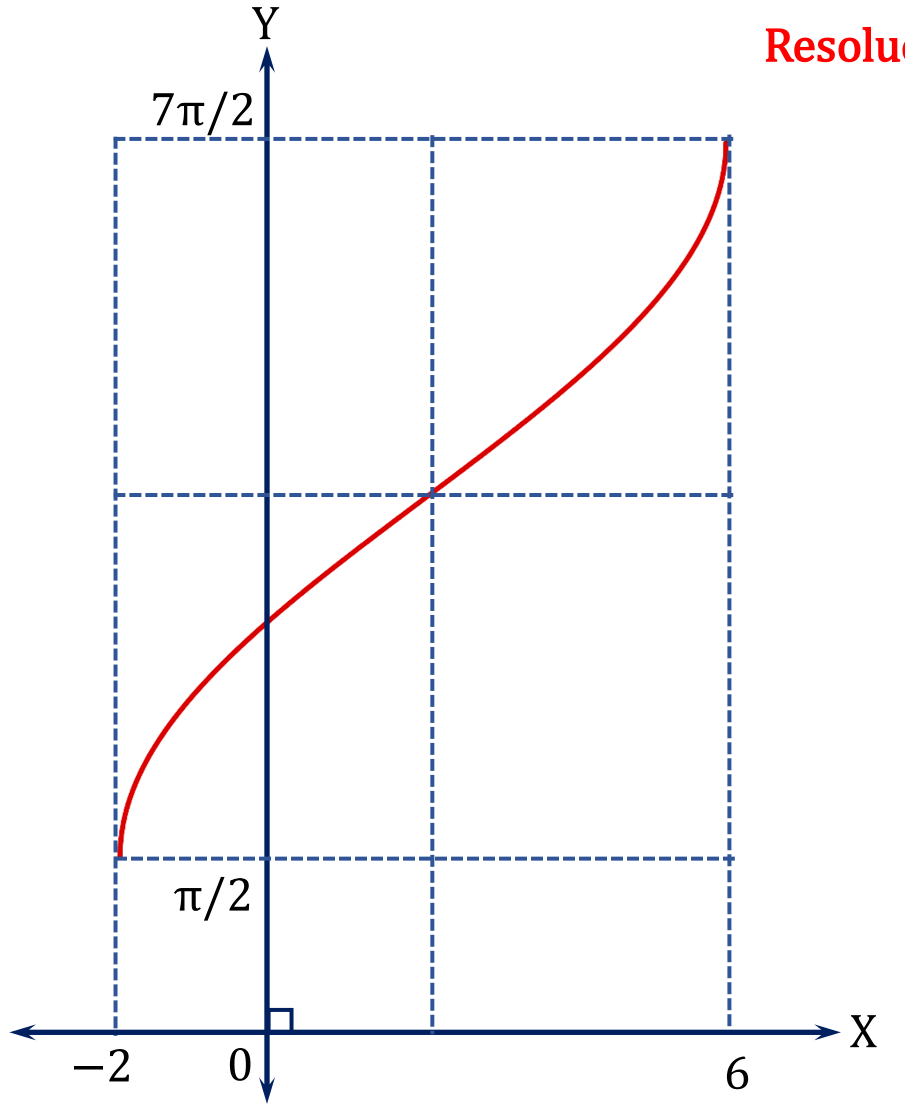
$$C = \frac{2\pi + 0}{2} \longrightarrow C = \pi$$

$$\therefore \mathbf{F(x) = 2arcSen(2x) + \pi}$$

CLAVE: C

FUNCIONES TRIGONOMÉTRICAS INVERSAS

19. Dado que la ecuación de la curva mostrada es: $y = A.\text{arcSen}(Bx + C) + D$,determinar: $(AD + 2B + C)$



Resolución:

$$A = \frac{\frac{7\pi}{2} - \frac{\pi}{2}}{\pi} \longrightarrow A = 3$$

$$B = \frac{2}{6 - (-2)} \longrightarrow B = \frac{1}{4}$$

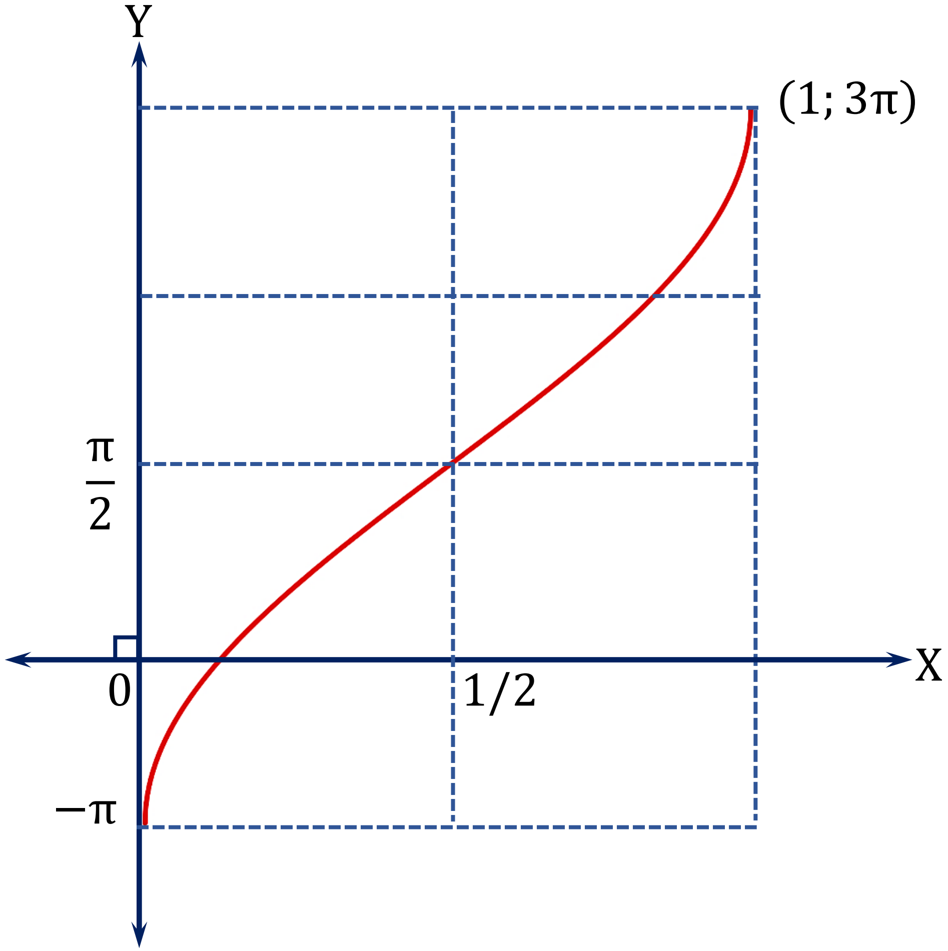
$$-\frac{C}{B} = \frac{6 + (-2)}{2} \longrightarrow -4C = 2 \longrightarrow C = -\frac{1}{2}$$

$$D = \frac{\frac{7\pi}{2} + \frac{\pi}{2}}{2} \longrightarrow D = 2\pi$$

$$3 \times 2\pi + 2\left(\frac{1}{4}\right) + \left(-\frac{1}{2}\right) \qquad \therefore 6\pi$$

CLAVE: E

20. Dado el gráfico de la función F, cuya regla de correspondencia es: $F(x) = A\text{Sen}^{-1}(Bx + C)$,determinar: $H = \frac{(A-B+C)}{D}\pi$



Resolución:

$$A = \frac{3\pi - (-\pi)}{\pi} \longrightarrow A = 4$$

$$B = \frac{2}{1-0} \longrightarrow B = 2$$

$$-\frac{C}{B} = \frac{1+0}{2} \longrightarrow -\frac{C}{2} = \frac{1}{2} \longrightarrow C = -1$$

$$D = \frac{3\pi + (-\pi)}{2} \longrightarrow D = \pi$$

$$H = \frac{(4 - 2 + (-1))}{\pi} \times \pi$$

$\therefore$ **H = 1**

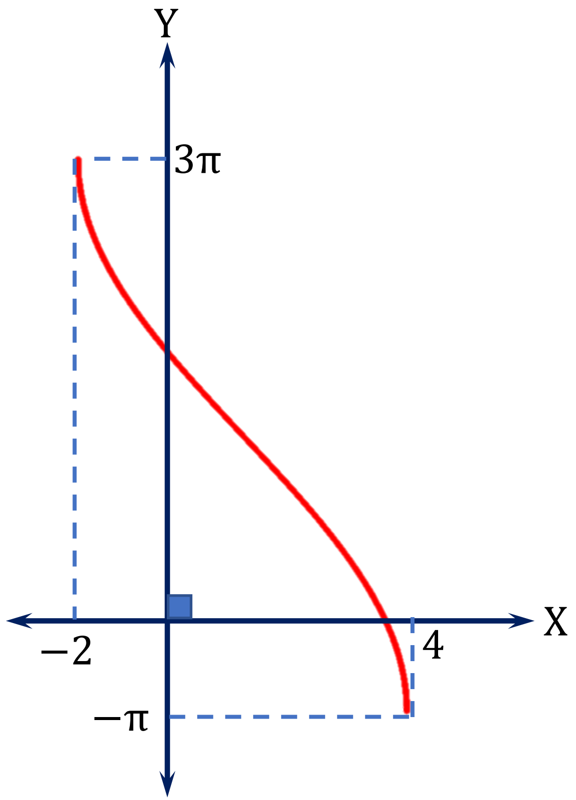
CLAVE: B

MOMENTO DE PRACTICAR

EXAMENES DE ADMISIÓN UNI

FUNCIONES TRIGONOMÉTRICAS INVERSAS

UNI 2020 – I Si la gráfica de $y = A.\operatorname{arcCos}(Bx + C) + D$ es



Determine el valor de $E = A + B + C$

A) 3 B) $\frac{2}{3}$ C) $\frac{4}{3}$

D) 4 E) $\frac{14}{3}$

Resolución:

$$A = \frac{3\pi - (-\pi)}{\pi} \longrightarrow A = 4$$

$$B = \frac{2}{4 - (-2)} \longrightarrow B = \frac{1}{3}$$

$$-\frac{C}{B} = \frac{4 + (-2)}{2} \longrightarrow -\frac{C}{B} = 1 \longrightarrow C = -\frac{1}{3}$$

$$E = A + B + C$$

$$E = 4 + \frac{1}{3} + \left(-\frac{1}{3}\right)$$

$\therefore$ **$E = 4$**

CLAVE: D

FUNCIONES TRIGONOMÉTRICAS INVERSAS

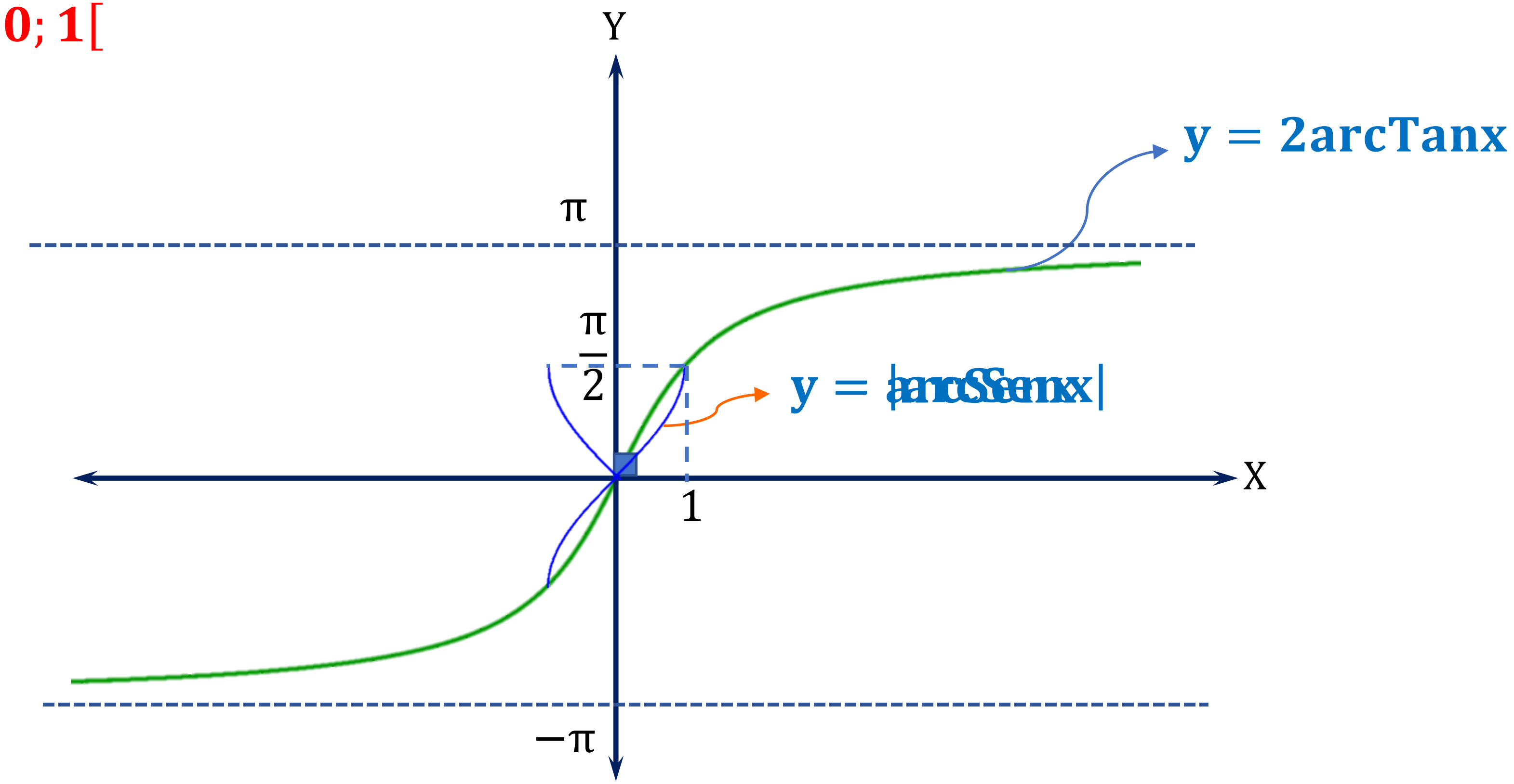
UNI 2018 – I Determine el conjunto solución de la inecuación: $|\text{arcSenx}| - 2\text{arcTanx} < 0$

Resolución:

$|\text{arcSenx}| < 2\text{arcTanx}$

Las gráficas se intersectan para $x = 1$

$\therefore \forall \mathbf{x} \in \mathbf{]0;1[}$



UNI 2017 – II

Simplifique la expresión: $H = \dfrac{\text{arcSen}\left(\dfrac{2a}{1+a^2}\right) + 2\text{arcCos}\left(\dfrac{1-a^2}{1+a^2}\right)}{\text{arcTan}[\text{arcCot}(\text{Tan}2a) - \text{arcCot}(\text{Tan}3a)]}$

Considerando $\text{arcTan}a \neq 0$

A) 2 B) 3 C) 4 D) 5 E) 6

Resolución:

Sea: $a = \text{Tan}\alpha$

$$H = \frac{\text{arcSen}\left(\dfrac{2\text{Tan}\alpha}{1+\text{Tan}^2\alpha}\right) + 2\text{arcCos}\left(\dfrac{1-\text{Tan}^2\alpha}{1+\text{Tan}^2\alpha}\right)}{\text{arcTan}\left[\cancel{\dfrac{\pi}{2}} - \text{arcTan}(\text{Tan}2a) - \left(\cancel{\dfrac{\pi}{2}} - \text{arcTan}(\text{Tan}3a)\right)\right]}$$

$$H = \frac{\text{arcSen}(\text{Sen}2\alpha) + 2\text{arcCos}(\text{Cos}2\alpha)}{\text{arcTan}[\text{arcTan}(\text{Tan}3a) - \text{arcTan}(\text{Tan}2a)]}$$

$$H = \frac{2\alpha + 2(2\alpha)}{\text{arcTan}a}$$

$$H = \frac{6\alpha}{\alpha} \quad \therefore \mathbf{H = 6}$$

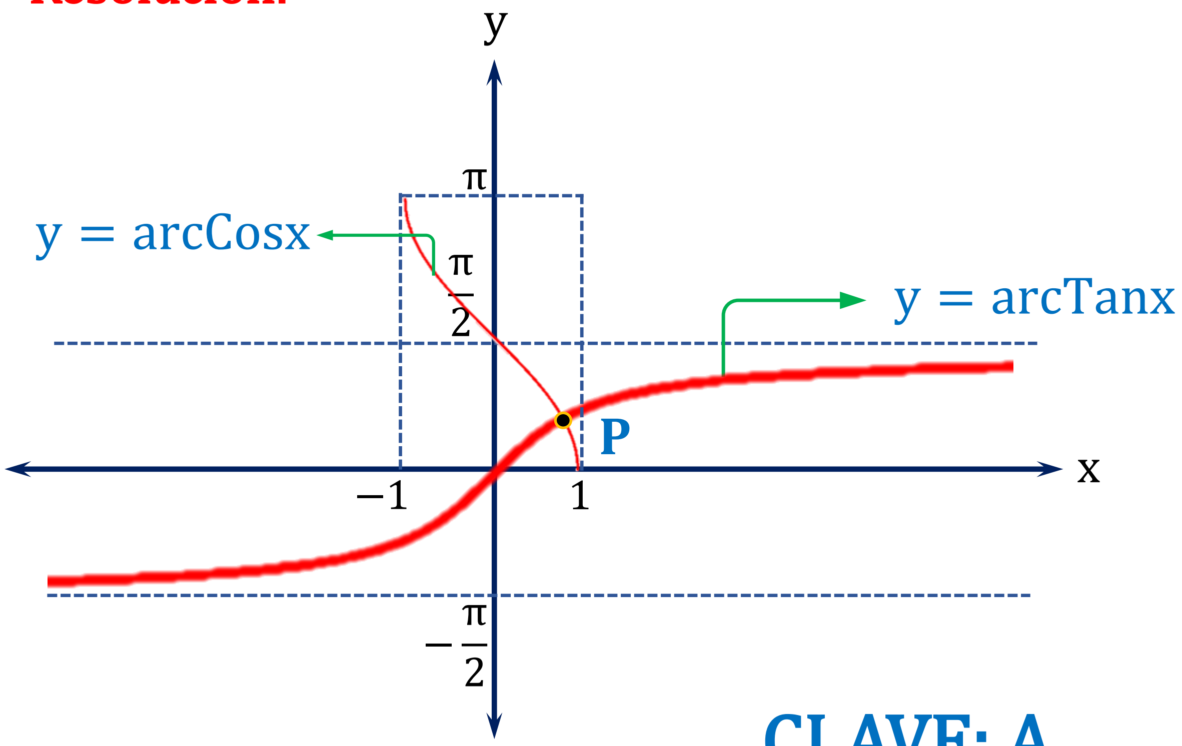
CLAVE: E

UNI 2017 – I

Las funciones arcCosx y arcTanx se intersectan en el punto P. Calcule la abscisa de P

A) $\frac{\sqrt{2\sqrt{5}-2}}{2}$ B) $\frac{\sqrt{2\sqrt{5}+2}}{2}$ C) $\frac{\sqrt{5}-1}{2}$ D) $\frac{\sqrt{5}-1}{4}$ E) $\frac{2\sqrt{5}+7}{2}$

Resolución:



CLAVE: A

$$\underbrace{\text{arcCos}x}_{\alpha} = \underbrace{\text{arcTan}x}_{\alpha}$$

$$\text{Cos}\alpha = x \wedge \text{Tan}\alpha = x$$

$$\text{Sec}\alpha = \frac{1}{x}$$

$$1 + \text{Tan}^2\alpha = \text{Sec}^2\alpha$$

$$1 + x^2 = \frac{1}{x^2}$$

$$x^4 + x^2 + \frac{1}{4} = 1 + \frac{1}{4}$$

$$\left(x^2 + \frac{1}{2}\right)^2 = \frac{5}{4}$$

$$x^2 + \frac{1}{2} = \pm\frac{\sqrt{5}}{2}$$

Como: $x^2 \geq 0$

$$x^2 = \frac{\sqrt{5}-1}{2}$$

Como la abscisa de P es positiva:

$$x = \sqrt{\frac{\sqrt{5}-1}{2}}$$

$$\therefore \mathbf{x = \frac{\sqrt{2\sqrt{5}-2}}{2}}$$

UNI 2017 – I

Dadas las funciones f y g definidas por: $f(x) = \arctan\left(\frac{2|x|}{1+x^2}\right), g(x) = \arcsin\left(\frac{x}{x^2+1}\right)$

Determine $\mathrm{Ran}(f) \cap \mathrm{Dom}(g)$.

A) $\left[0; \frac{\pi}{4}\right]$ B) $\mathbb{R}$ C) $]-\infty; 1]$ D) $[1; +\infty[$ E) $[0; 1]$

Resolución:

$$f(x) = \arctan\left(\frac{2|x|}{1+x^2}\right)$$

$$\frac{2|x|}{1+x^2} = \frac{2}{\frac{1+|x|^2}{|x|}} = \frac{2}{|x| + \frac{1}{|x|}}$$

$$|x| + \frac{1}{|x|} \geq 2$$

$$0 < \frac{1}{|x| + \frac{1}{|x|}} \leq \frac{1}{2}$$

$$0 < \frac{2}{|x| + \frac{1}{|x|}} \leq 1$$

$$0 < \arctan\left(\frac{2}{|x| + \frac{1}{|x|}}\right) \leq \frac{\pi}{4}$$

Para $x = 0$, f está definida

$$\arctan\left(\frac{2|x|}{1+x^2}\right) = 0$$

$$\mathrm{Ran}(f) = \left[0; \frac{\pi}{4}\right]$$

$$g(x) = \arcsin\left(\frac{x}{x^2+1}\right)$$

$$\frac{x}{x^2+1} = \frac{1}{\frac{x^2+1}{x}} = \frac{1}{x + \frac{1}{x}}$$

$$x + \frac{1}{x} \geq 2 \lor x + \frac{1}{x} \leq -2$$

$$0 < \frac{1}{x + \frac{1}{x}} \leq \frac{1}{2} \lor -\frac{1}{2} \leq \frac{1}{x + \frac{1}{x}} < 0$$

$$-\frac{1}{2} \leq \frac{1}{x + \frac{1}{x}} \leq \frac{1}{2}$$

Para $x = 0$, g está definida

$$-\frac{1}{2} \leq \frac{1}{x + \frac{1}{x}} \leq \frac{1}{2} \subset [-1; 1]$$

$$\mathrm{Dom}(g) = \mathbb{R}$$

$$\mathrm{Ran}(f) \cap \mathrm{Dom}(g)$$

$$\left[0; \frac{\pi}{4}\right]$$

CLAVE: A

UNI 2016 – I

Determine el rango de la función $f: [-1; 1] \longrightarrow \mathbb{R}$ definida por: $f(x) = \dfrac{\text{arcSenx} + \frac{\pi}{2}}{\text{arcCosx} - 2\pi}$

A) $[-1; 0]$ B) $\left[-\frac{1}{2}; 0\right]$ C) $\left]-\frac{1}{2}; \frac{1}{2}\right[$ D) $\left[-\frac{1}{2}; \frac{1}{2}\right]$ E) $[0; 1]$

Resolución:

$$f(x) = \left(\frac{\text{arcSenx} + \frac{\pi}{2}}{\text{arcCosx} - 2\pi} + 1\right) - 1$$

$$f(x) = \frac{\text{arcSenx} + \text{arcCosx} + \frac{\pi}{2} - 2\pi}{\text{arcCosx} - 2\pi} - 1$$

$$f(x) = \frac{\frac{\pi}{2} + \frac{\pi}{2} - 2\pi}{\text{arcCosx} - 2\pi} - 1$$

$$f(x) = \frac{-\pi}{\text{arcCosx} - 2\pi} - 1$$

- $0 \leq \text{arcCosx} \leq \pi$

$$-2\pi \leq \text{arcCosx} \leq -\pi$$

$$-\frac{1}{2\pi} \leq \frac{1}{\text{arcCosx}} \leq -\frac{1}{\pi}$$

$$\frac{1}{2} \leq \frac{-\pi}{\text{arcCosx}} \leq -1$$

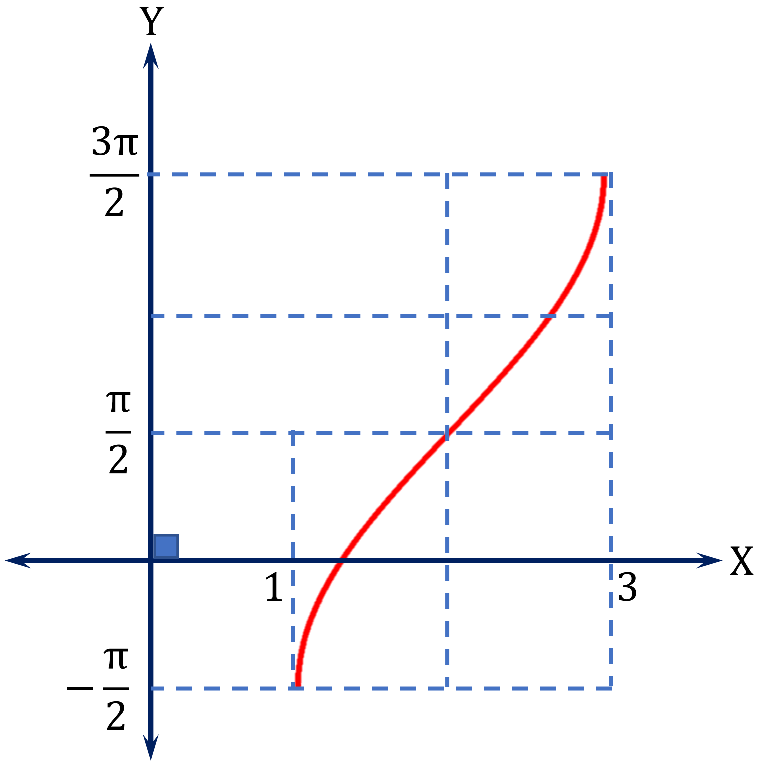
$$\frac{1}{2} - 1 \leq \frac{-\pi}{\text{arcCosx}} - 1 \leq -1 - 1$$

$$-\frac{1}{2} \leq f(x) \leq 0$$

$$\therefore \mathbf{f(x)} \in \left[-\frac{1}{2}; 0\right]$$

CLAVE: B

UNI 2015 – II Sea la función $y = A.\text{arcSen}(Bx + C) + D$; $A, B > 0$ con gráfica



Calcule $K = A + B + C\left(\frac{4D}{\pi}\right)$

A) -2 B) -1 C) 0
D) 2 E) 4

Resolución:

$$A = \frac{\frac{3\pi}{2} - \left(-\frac{\pi}{2}\right)}{\pi} \longrightarrow A = \frac{\frac{3\pi}{2} + \frac{\pi}{2}}{\pi} \longrightarrow A = 2$$

$$B = \frac{2}{3-1} \longrightarrow B = 1$$

$$-\frac{C}{B} = \frac{3+1}{2} \longrightarrow -\frac{C}{B} = 2 \longrightarrow C = -2$$

$$D = \frac{\frac{3\pi}{2} + \left(-\frac{\pi}{2}\right)}{2} \longrightarrow D = \frac{\frac{3\pi}{2} - \frac{\pi}{2}}{2} \longrightarrow D = \frac{\pi}{2}$$

$$K = A + B + C\left(\frac{4D}{\pi}\right)$$

$$K = 2 + 1 + (-2)\left(\frac{4\left(\frac{\pi}{2}\right)}{\pi}\right)$$

$\therefore$ **$K = -1$**

CLAVE: B

UNI 2015 – I Sea la función $f(x) = \dfrac{x^3}{\operatorname{arcTan}(x) - x}$

Dadas las siguientes proposiciones:

I. La función es impar

II. Si $x \in \operatorname{Dom}(f)$, entonces $-x \in \operatorname{Dom}(f)$

III. La gráfica de f corta la curva $y = x^2$

Son correctas:

A) Solo I B) Solo II C) Solo III D) I y II E)II y III

Resolución:

$$f(x) = \frac{x^3}{\operatorname{arcTan}(x) - x}$$

$$\operatorname{Dom}(f) = \mathbb{R} - \{0\}$$

I. La función es impar ✗

$$f(-x) = \frac{(-x)^3}{\operatorname{arcTan}(-x) - (-x)}$$

$$f(-x) = \frac{x^3}{-\operatorname{arcTanx} - x}$$

$$f(-x) = f(x)$$

Función Impar Una función f es impar si se cumple que:

$f(-x) = -f(x), \forall\, x\,; -x \in \operatorname{Dom}(f)$

Función Par Una función f es par si se cumple que:

$f(-x) = f(x), \forall\, x\,; -x \in \operatorname{Dom}(f)$

II. Si $x \in \operatorname{Dom}(f)$, entonces $-x \in \operatorname{Dom}(f)$ ✓

Como la función es par, si $x \in \operatorname{Dom}(f)$

Entonces $-x \in \operatorname{Dom}(f)$

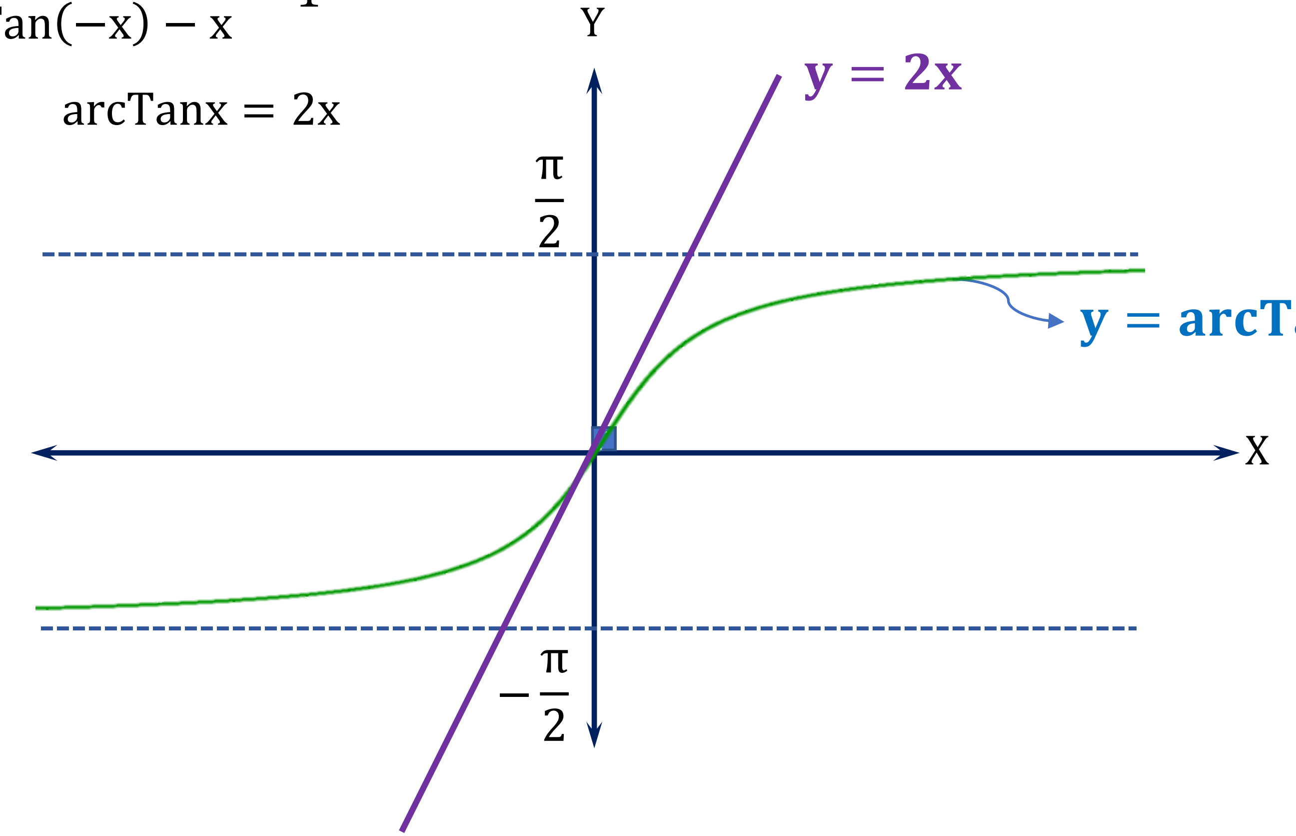
III. La gráfica de f corta la curva $y = x^2$

Supongamos que f corta a la curva $y = x^2$, entonces $f(x) = x^2, x \neq 0$

$$\frac{x^3}{\text{arcTan}(-x) - x} = x^2 \quad ✗$$

$$\frac{x}{\text{arcTan}(-x) - x} = 1$$

$$\text{arcTan}x = 2x$$



Observamos que si $\text{arcTan}x = 2x$, entonces $x = 0$ pero $x \neq 0$

Por lo tanto, las gráficas de f y de $y = x^2$ no se cortan

∴ Solo II es correcta

CLAVE: B

FUNCIONES TRIGONOMÉTRICAS INVERSAS

UNI 2014 – I Si $x \in]-\infty; 0[$, entonces el rango de la función $f(x) = \dfrac{5\pi}{|\text{arcTan}x| + 2|\text{arcCot}x|}$, es :

A) $]0; 1[$ B) $]1; 2[$ C) $]0; 2[$ D) $]2; 5[$ E) $]5; +\infty[$

Resolución:

$$x < 0 \longrightarrow \begin{array}{l} |\text{arcTan}x| = -\text{arcTan}x \\ |\text{arcCot}x| = \text{arcCot}x \end{array}$$

$$f(x) = \frac{5\pi}{|\text{arcTan}x| + 2|\text{arcCot}x|}$$

$$f(x) = \frac{5\pi}{-\text{arcTan}x + 2\text{arcCot}x}$$

$$f(x) = \frac{5\pi}{-\left(\frac{\pi}{2} - \text{arcCot}x\right) + 2\text{arcCot}x}$$

$$f(x) = \frac{5\pi}{3\text{arcCot}x - \frac{\pi}{2}}$$

Cuando $-\infty < x < 0$, se forma la función f(x)

$$\frac{\pi}{2} < \text{arcCot}x < \pi$$

$$\frac{3\pi}{2} < 3\text{arcCot}x < 3\pi$$

$$\frac{3\pi}{2} - \frac{\pi}{2} < 3\text{arcCot}x - \frac{\pi}{2} < 3\pi - \frac{\pi}{2}$$

$$\pi < 3\text{arcCot}x - \frac{\pi}{2} < \frac{5\pi}{2}$$

$$\frac{1}{\pi} < \frac{1}{3\text{arcCot}x - \frac{\pi}{2}} < \frac{2}{5\pi}$$

$$5 > \frac{5\pi}{3\text{arcCot}x - \frac{\pi}{2}} > 2$$

$$5 > f(x) > 2$$

$\therefore\ \mathbf{f(x) \in\ ]2; 5[}$

CLAVE: D

UNI 2013 – II Halle el dominio de la función $f(x) = 17\text{arcSec}\left(x - \frac{3}{2}\right)$

A) $\left]-\infty; -\frac{1}{2}\right] \cup \left[\frac{5}{2}; +\infty\right[$

B) $\left]-\infty; \frac{1}{2}\right] \cup \left[\frac{5}{2}; +\infty\right[$

C) $\left]-\infty; -\frac{3}{2}\right] \cup \left[\frac{1}{2}; +\infty\right[$

D) $\left]-\infty; -\frac{1}{2}\right] \cup \left[\frac{1}{2}; +\infty\right[$

E) $\left]-\infty; -\frac{5}{2}\right] \cup \left[\frac{3}{2}; +\infty\right[$

Resolución:

$$f(x) = 17\text{arcSec}\left(x - \frac{3}{2}\right)$$

$$x - \frac{3}{2} \leq -1 \quad \vee \quad 1 \leq x - \frac{3}{2}$$

$$x \leq \frac{1}{2} \quad \vee \quad \frac{5}{2} \leq x$$

$$\therefore \textbf{Domf} = \left]-\infty; \frac{1}{2}\right] \cup \left[\frac{5}{2}; +\infty\right[$$

CLAVE: B

UNI 2013 – I

Señale la alternativa que presenta la secuencia correcta, después de determinar si la proposición es verdadera (V) o falsa (F):

I. Si $\text{arcSen}(-x) = -\frac{\pi}{2}$, entonces $x = 1$

II. Si $\text{arcCos}(-x) = 1$, entonces $x = -\pi$

III. Si $x \in [-1; 1]$, entonces $\text{arcSen}(-x) + \text{arcCos}(-x) = \frac{\pi}{2}$

A) FFV B) VVV C) VVF D) VFF E)VFV

Resolución:

I. Si $\text{arcSen}(-x) = -\frac{\pi}{2}$, entonces $x = 1$ **(V)**

$$\text{arcSen}(-x) = -\frac{\pi}{2}$$

$$x = \text{Sen}\left(-\frac{\pi}{2}\right)$$

$$-x = -1$$

$$\mathbf{x = 1}$$

II. Si $\text{arcCos}(-x) = 1$, entonces $x = -\pi$ **(F)**

$$\text{arcCos}(-x) = 1$$

$$-x = \text{Cos}(1) \qquad \mathbf{x = -Cos(1)}$$

III. Si $x \in [-1; 1]$, entonces $\text{arcSen}(-x) + \text{arcCos}(-x) = \frac{\pi}{2}$ **(V)**

$$x \in [-1; 1] \longrightarrow -x \in [-1; 1]$$

Por propiedad:

$$\text{arcSen}(-x) + \text{arcCos}(-x) = \frac{\pi}{2}$$

$\therefore$ **VFV**

CLAVE: E

UNI 2011 – II

Para 0<x<1, resuelva la ecuación $\operatorname{arcCot}x = \operatorname{arcTan}\left(\frac{1}{\sqrt{1-x}}\right)$

A) $\frac{-1+\sqrt{5}}{2}$ B) $\frac{-1+\sqrt{4}}{2}$ C) $\frac{-1+\sqrt{3}}{2}$ D) $\frac{-1+\sqrt{2}}{2}$ E) $\frac{-2+\sqrt{2}}{2}$

Resolución:

De la condición:

$$\operatorname{arcCot}x = \operatorname{arcTan}\left(\frac{1}{\sqrt{1-x}}\right); 0 < x < 1$$

$$\operatorname{arcTan}\left(\frac{1}{x}\right) = \operatorname{arcTan}\left(\frac{1}{\sqrt{1-x}}\right)$$

$$\frac{1}{x} = \frac{1}{\sqrt{1-x}}$$

$$\frac{1}{x^2} = \frac{1}{1-x}$$

$$x^2 = 1 - x$$

$$ax^2 + bx + c = 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x^2 + x - 1 = 0$$

$$x = \frac{-1 \pm \sqrt{(-1)^2 - 4(1)(-1)}}{2(1)}$$

$$x = \frac{-1 \pm \sqrt{5}}{2}$$

$$x = \frac{-1 + \sqrt{5}}{2} \checkmark \quad \vee \quad x = \frac{-1 - \sqrt{5}}{2} \times$$

$$\therefore \mathbf{x = \frac{\sqrt{5} - 1}{2}}$$

CLAVE: A

FIN DE LA SESIÓN

PRACTICA Y APRENDERÁS